4.2. Solving differential models based on ordinary differential equations by the Runge-Kutta method

Runge-Kutta method can be used to obtain approximate solutions (solution of the Cauchy problem) ordinary differential equations. It requires more computing performance action than Euler method, but has fourth order of accuracy relative step calculation nodes.

Let the first order differential equation in the form:

y '= f (x, y),

with initial conditions y (xo) = yo, on the interval from x0 to x hn.

We divide the segment in question on the part of the step h. Then discrete coordinate can be found on dependence:,

where i = 0, 1, 2 ... n - node number estimated net.

According to the usual method of Runge-Kutta, analytic function y (x), which are to be replaced by its discrete analog - table function VI, which with sufficient accuracy replaces analytical. Successive values VI + 1 functions are defined by the expression:

VI VI = + 1 + Δui + εi,

where εi = VI + 1 - y (xi + 1) - nodular error method Runge-Kutta;

Δui, = 1/6 · (Au + + 2Vi 2Si + Di).

The coefficients Ai, Bi, Cu, Di - features that every step defined as follows:

Ai = h · f (x, y);

Bi = h · f (x + h / 2, y + Ai / 2);

Ci = h · f (x + h / 2, y + Bi / 2);

Di = h · f (x + h / 2, y + Ci).

In these formulas as the right of f (x, y) differential equation can also serve its discrete analog f (xi, yi). Then they look like:

Ai = h · f (xi, yi);

Bi = h · f (xi + h / 2, yi + Ai / 2);

Ci = h · f (xi + h / 2, yi + Bi / 2);

Di = h · f (xi + h / 2, yi + Ci).

The error that accumulated throughout a predetermined interval.

If | εi | and | ε | very small compared with the actual values of the VI and do not affect the calculation of the final interpretation of the results, then the approximate solution in the form of a table function is considered convergent VI to the analytical dependence y (x) in the appropriate units and. So for recurrent calculations using formula Runge-Kutta as:

VI VI = + 1 + Δui.

Considered normal kung Kutta method has fourth order of accuracy relative step. This means that εi ~ h4. That is, increasing half step (for example, in order to save computing resources), the error εi increases 32 times and vice versa. Thus, the method responds very hard to change a step in terms of sustainability. Example. Differential equations of mathematical models to determine the amount of substance x C obtained at any time t> 0 in the reaction of the two starting materials A and B, has the following differential equations. ,

where - coefficient of reaction rate, which may depend on the reaction temperature, concentration of ingredients, other factors and approximated dependence on time t; - amount of substance A and B which react, respectively.β and α

C:χ A and volume matter get in the total volume of substance β αIt should also be noted that the volume of substances

.β + α = χ

The initial conditions determine the amount of C0 C substances obtained for any given specific time t0:

x (t0) = x0.