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MODELING AND CALCULATION OF TECHNOLOGICAL PROCESSES

Technique of Parametric and Heat Computations of Rollers for Processing of Plastics and Rubber Compounds

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Abstract— The technique of parametric and thermal simulation of rollers of continuous operation was developed for processing material whose behavior under load is described by a power rheological law. The technique is suitable for analyzing the process of rolling in the case of rollers with the same diameter, arbitrary frictions in a roll space, and also placing of rolled material in a low-speed as well as in a high-speed rollers.

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One of the preliminary treatments of polymeric materials and rubber compounds that determines the quality of products derived from them, is rolling, the process of repeatedly punching of molding sand through the clearance between two parallel oppositely rotating rollers leading to its heating, mixing and homogenization.

Simultaneous introduction in the roller space gap of polymer or rubber, as well as various solid and liquid components can produce a high-quality material, which as a result of temperature adjustment of rollers and their speeds sticks to one of them [1, 2]. Typically, the material sticks to a hotter roller, and in the case of identical temperatures of rollers, to more high-speed one.

Classification of rollers is carried on a number of specific design or manufacturing features [3, 4], and one of the main characteristics of the rollers is frictions: a ratio of peripheral speeds of adjacent rollers (usually the friction f is the ratio of the peripheral speeds of high-speed and low-speed rollers and the friction value is not less than unity. A reverse value of friction is called the coefficient of frictions ψ [5, 6]).

Nowadays there is no a calculation technique for the rolling in the case of the arbitrary friction in the roll space and also if a material being processed is located both at the front and rear rollers.

Purpose of the study is developing calculation technique for the rolling of a material whose behavior is described by the power rheological equation at the arbitrary friction into the roll space, and also of a roller on which there is a material being processed in the course of the rolling.

SIMULATION OF THE ROLLING

A blending cycle on rollers of periodic (cyclic) action depends on the rheological and thermal properties of the treated material and on parameters of the rolling and is finished provided achievement of a certain degree of homogeneity or the required temperature. On more productive rollers of the continuous action the rolling time depends on the speed of the rollers, the distance between a place of feeding the material and a discharging place on a next stage of processing (e.g., calendering), a width of a continuous strip of material withdrawn from the rollers and also of a roller with processed material on it.

Feeding the material processed into blending continuous rollers after a batch blender of closed type (e.g., such as "Banbury") is carried out in the form of shapeless lumps, and into the mix-heating continuous rollers after the blending rollers, in the form of a continuous strip. Moreover, the feeding is usually

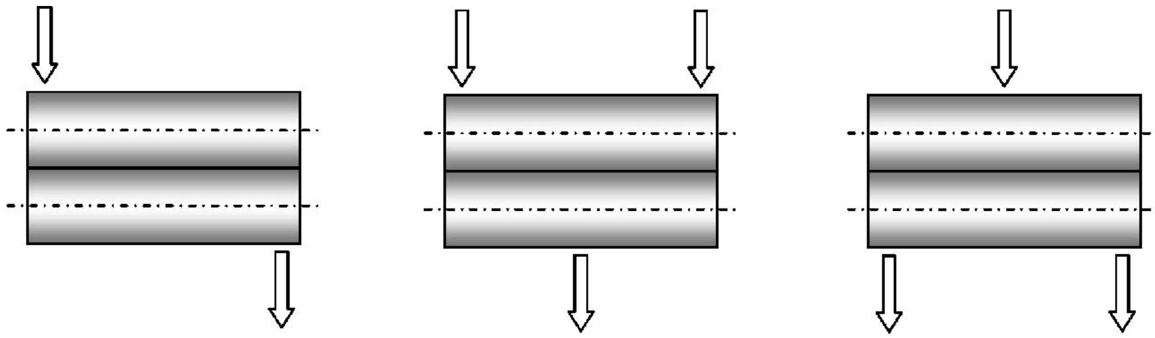

Fig. 1. Examples of feeding the material into rollers of continuous operation and discarding the material from them.

carried out from one end of the roll body, and the discarding, at the other (Fig. 1a). In some cases, feeding the material into the rollers is carried out on both ends of the roll space (Fig. 1b) and discarding, at the middle, or feeding via the center of the roll space, and discarding at both its ends (Fig. 1c). Removal of the final composition in the form of the continuous strip, which is separated from the rolled material by special knives, is performed usually in the direction of an adjustment mechanism of a value of the roll space.

Scheme of the continuous rollers, which is fed from one side of the rolls, and discharged at the opposite is shown in Fig. 2. This scheme, unlike others, provides a maximum time of rolling, and thus the best quality of the composition. After leaving the roll space at a loading section the material being rolled sticks to one of the rollers: the front (low speed) (Fig. 2a) or rear (high-speed) (Fig. 2b), and returns to the roll space spreading along it. Further, this sequence is repeated. At each turn of the roller the material gradually moves in a spiral to a place of its discarding from the rollers.

Thus, in continuous rolling the material passes successively sections of the roll space, which alternate with zones of contact of the material with a roller heated to a certain temperature on the one hand and the environment (air) on the other. The temperature of the material gradually increases. In order to intensify the mixing it is reasonable to increase a number of passes of the material through the roll space (but then the roll productivity is reduced and the likelihood of a thermal destruction of components of the material increases).

The purpose of the technique of the continuous roll calculation is to determine the energy-power parameters of the rolling, and its main tasks is to determine:

(1) The temperature field of the material being rolled in the course of its movement from the place of its feeding into the rollers to the place of its removal from the rollers;

(2) A spacer effort acting on the rolls;

(3) A power consumed by the drive of the rollers;

(4) Parameters of a cooling agent in the rollers (temperature and flow rate).

The number of the material passes k via the roll space is: for scheme in Fig. 1a $k = L/b$; for scheme in Figs. 1b and 1c $k = L/2b$, where L is a distance between the restrictive arrows of rollers (a working length of the roll body), m; b, a width of the continuous strip of the material withdrawn from the rollers, m.

The rolling time of the material is calculated by the following equation

$$t = \frac{2\pi R_r k}{W_{h(l)}},$$

where R_r is a radius of the roll body.

Mass productivity of the rolls G (kg s^{-1}) is

$$G = \rho b \delta W_{h(l)},$$

where ρ is the material density, kg m^3; δ is a thickness of the material strip, withdrawn from the rolls, m.

Mathematical model of the flow of the rolled material in the roll space contains the differential equations of continuity, motion, energy, rheological equation, and the condition of uniqueness [1, 5, 6].

In the course of development of the mathematical model we accounted for the following assumptions (Fig. 3):

– the material processed is incompressible;

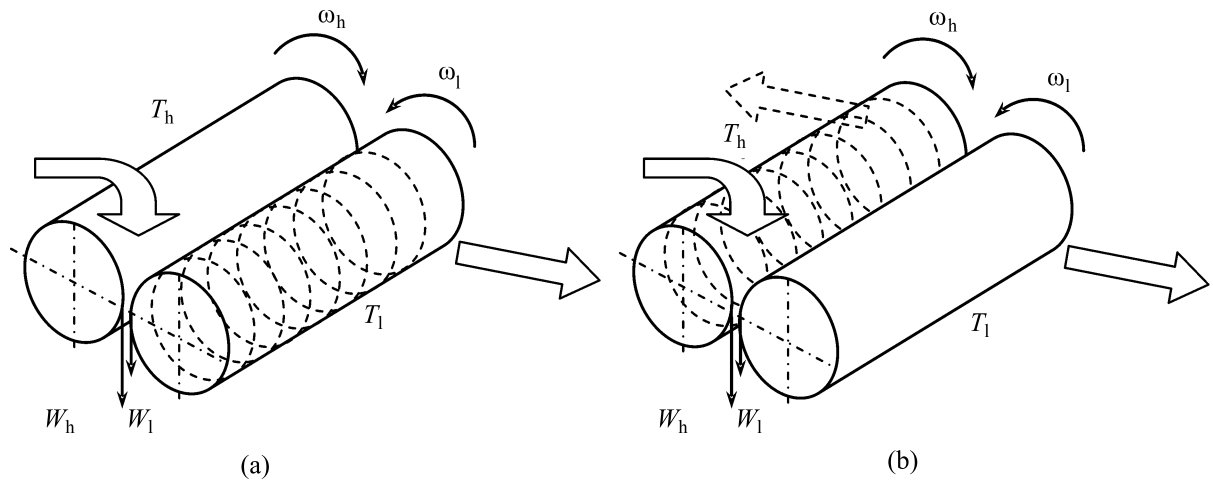

Fig. 2. Scheme of the continuous rolling in the case of location of the material processed (a) on the low speed roller and (b) on the high speed roll. (ω_h, ω_l) the angular velocities of the high-speed and low-speed rollers; (W_h, W_l) linear velocities of the operational surfaces of the high-speed and low-speed rollers; (T_h, T_l) temperatures of the operational surfaces of the high-speed and low-speed rollers (arrows show ways of supporting and removal of the material).

– overpressure at the beginning and end of the zone of deformation of the roll space is zero;

– weight of the material in the zone of deformation of the roll space is small compared to the pressure, so they are neglected;

– in the case of a contact with the surface of the roll the material processed sticks to it;

– the roll space is small compared with the radius and a working length of the roller;

– acceleration of the material is small due to a high viscosity and low rolling speed;

– rheological and thermal properties of the material depend on temperature;

– heat transfer along the roll space by thermal conductivity is neglected.

Accounting for the above assumptions the differential equations describing the process of the material flow in the roll space take the form:

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} = 0; \tag{1}$$

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{dy} = 0; \tag{2}$$

$$\rho c_p w_x \frac{\partial T}{\partial x} = -\frac{\partial q_y}{\partial y} + q_{diss}; \tag{3}$$

$$\tau_{xy} = K\left|\frac{\partial w_x}{\partial y}\right|^n \operatorname{sgn}\left(\frac{\partial w_x}{\partial y}\right), \tag{4}$$

where w_x and w_y are components of the material speed along the x and y axes, m s^{-1} (Fig. 3); p, pressure in the roll space, Pa; τ_{xy}, shear stress, Pa; T, temperature, K; ρ and c_p, density (kg m^{-3}) and mass heat capacity (J kg^{-1} K^{-1}) of the material processed as function of the temperature; q_y, heat flux in the direction of y, W m^{-2}; q_{diss}, an intensity of dissipation energy, W m^{-3}; n, exponent of rheological equation.

Dependence of a consistency K on temperature T is given by equation

$$K = K_0 \exp\left(-\beta \frac{T - T_0}{T_0 + 273}\right), \tag{5}$$

wher K_0 is consistency (Pa s^{-n}), determined at temperature T_0, K; β, temperature coefficient of the rheological equation.

The initial condition for temperature

$$T|_{x = x_{ini}} = T_{ini}(y). \tag{6}$$

Boundary conditions (Fig. 3):
– for velocity

$$w_x|_{y=-h} = W_l = \psi W_h; \tag{7}$$

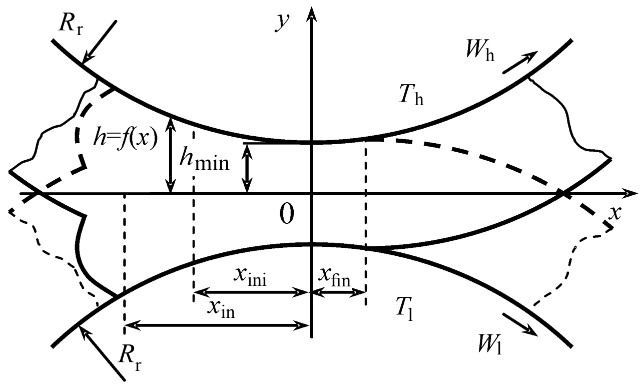

Fig. 3. Scheme of the material flow in the roll space. (x, y) Coordinates directed along and across the roll space; (x_{in}, x_{ini}, x_{fin}) inlet coordinates of the material in the roll space, coordinates of starting and final sections of deformation; (h, h_{min}) half of the current and minimum roll space.

$$w_x|_{y=h} = W_h, \quad (8)$$

– for temperature

$$T|_{y=-h} = T_l; \quad (9)$$

$$T|_{y=h} = T_h, \quad (10)$$

where $\psi = W_l/W_h$. Set of Eqs. (1)–(5) is solved by introducing dimensionless variables ξ and ε [1, 6]:

$$\xi = \frac{x}{\sqrt{2R_r h_{min}}}\ ; \ \varepsilon = \frac{y}{h}.$$

where $h \approx h_{min} + \dfrac{x^2}{2R_r} = h_{min}\left(1+\xi^2\right)$

The set of Eqs. (1)–(5) for the initial (6) and boundary (7)–(10) conditions enables calculating:
– The temperature field of the material in any section of the roll space;
– Forces acting on the rolls;
– Torques acting on the rolls and, thus, the drive power of the rolls;
– Value of the dissipation energy, being got by the material as a result of irreversible shear strain into the roll space.

A differential equation of unsteady heat conductivity written in cylindrical coordinates, under certain initial and boundary conditions should be solved to determine the temperature distribution of material when latter is out of bounds of the roll space.

In order to simplify the original equation we assumed:
– Movement of layers of the material relative to each other is not available;
– Heat transfer is performed uniformly axially along the radius of the roller due to heat conduction. Heat transfer along the axis of the roller is neglected;
– The size of the material located on the roll out of the bounds of the roll space does not change;
– The material sticks to the roll surface;
– Heat transfer in the material by heat conduction is performed according to the Fourier law

$$q_r = \lambda \frac{\partial T}{\partial r},$$

where q_r is heat flux along the axis r, which coincides with the radius of the roller, W m^{-2}; λ, heat conductivity of the material as a function of the temperature, W m^{-1} K^{-1}.

In view of the assumptions the equation of unsteady heat conductivity takes the form

$$\rho c_p \frac{\partial T}{\partial t} = -\frac{\partial}{\partial r}\left(\lambda \frac{\partial T}{\partial r}\right) + \frac{\lambda}{r}\frac{\partial T}{\partial r}, \quad (11)$$

where t is the time, s.

The initial condition is the temperature distribution of the material at the inlet to section under consideration corresponding to the temperature distribution at the outlet from the roll space at a current turnover of the material on the roller:

$$T|r = T(r). \tag{12}$$

Temperature boundary conditions:

$$T|r = R_r = T_{h(l)}; \tag{13}$$

$$\lambda \frac{\partial T}{\partial r}\Big|_{r=R_r+\delta} = \alpha \left(T|_{r=R_r+\delta} - T_{env}\right), \tag{14}$$

where α is the coefficient of heat transfer from the surface of the material to the environment; T_{env}, ambient temperature.

The solution of Eq. (11) in the case of initial (12) and boundary (13), (14) conditions enables calculation of the temperature field of the material during the heat exchange of the material to roller with one hand and to the environment on the other.

Now we consider the technique of the parametric and thermal design of rollers.

COMPUTATION OF PARAMETERS OF THE ROLL SPACE

The sequence of calculating the value of the roll space $2h_{min}$, the thickness of a continuous strip δ withdrawn from rollers, the velocities of the roll body surfaces W_h and W_l, as well as the mass performance of the rolls G depend on the source data. In this case these quantities are linked as follows.

Peripheral velocity $W_{h(l)}$ of the roll body surface from which the continuous strip of material is removed, is preset or computed by the dependence

$$W_{h(l)} = \frac{G}{\rho b \delta}.$$

Then the velocity of the roll body surface free from the material processed is: $W_l = \psi W_h$ or $W_h = W_l/\psi$.

The thickness of the molded material after the roll space is as follows:

– for the high-speed roller

$$\delta = h_{min}(1 + \psi)(1 + \xi_{fin}^2).$$

– for the low-speed roller

$$\delta = h_{min}\frac{(1 + \psi)}{\psi}(1 + \xi_{fin}^2).$$

The boundaries of the deformation zones are the roll surfaces and cross-sections of the roll space in which the overpressure in the material processed is zero. In the assumed system of dimensionless coordinates ξ and ε the surface of the rollers corresponds to the coordinates ε = ±1, and the the cross-sections corresponding to the beginning and end of the deformation zone, to coordinates ξ_{ini} ξ_{fin}.

Coordinate ξ_{fin} is determined experimentally. Its value is typically in a range $|\xi_{fin}| = 0.2...0.4$ [6].

Coordinate ξ_{ini} corresponds to a value of coordinate ξ, at which the following equality [5, 6]

$$\int_{\xi_{fin}}^{\xi_{ini}} \left[\frac{|A|^n \operatorname{sgn}(A) - |B|^n \operatorname{sgn}(B)}{1+\xi^2}\right] d\xi = 0,$$

where

$$A = \left(\frac{1+2n}{n}\right)\frac{(1+\psi)\left(\xi^2 - \xi_{fin}^2\right)}{\left(1+\xi^2\right)^2} + \frac{1-\psi}{1+\xi^2},$$

$$B = -\left(\frac{1+2n}{n}\right)\frac{(1+\psi)\left(\xi^2 - \xi_{fin}^2\right)}{\left(1+\xi^2\right)^2} + \frac{1-\psi}{1+\xi^2}.$$

One of the characteristics of the roll space is an inlet coordinate ξ_{in} of the material into the space determining the position of the free surface of a rotating allowance whose width is

$$2h_{in} = 2h_{min}(1 + \xi_{in}^2).$$

Then coordinate ξ_{in} is

$$\xi_{in} = \sqrt{\frac{h_{in}}{h_{min}} - 1}.$$

To ensure sufficient heating of the material at the inlet to the roll space, as well as its stirring and homogenization a ratio $(2h_{in})/(2h_{min})$ should be in a range of 5 to 20, which corresponds to the coordinate ξ_{in} from 2.0 to 4.5 [6].

Angles which correspond to coordinates of the material input in the roll space and output from the space counted from plane passing through the longitudinal axis of the rollers should be known to calculate the temperature field of the material from the place of loading material to the place of its discharging from the rollers, as well as to determine the heat loss from the surface of the material and the free surface of the rollers.

Let us denote the angle corresponding to the coordinate of material inlet into the roll space as $\gamma_{\xi_{in}}$ and angle corresponding to the material output from the space $\gamma_{\xi_{fin}}$. Then, we can write

$$\gamma_{\xi_{in}} = \arcsin\frac{\xi_{in}\sqrt{2R_r h_{min}}}{R_r},$$
$$\gamma_{\xi_{fin}} = \arcsin\frac{\xi_{fin}\sqrt{2R_r h_{min}}}{R_r}.$$

Central angles corresponding to the free surface of jth roll $j = (1, 2)$ and the roll surface covered by the material (γ_{freej} and γ_{mj}, respectively) are computed according the following dependences

$$\gamma_{free1} = 360° - \gamma_{\xi in} - \gamma_{\xi fin};\ \gamma_{free2} = 0;\ \gamma_{m1} = 0;$$
$$\gamma_{m2} = 360° - \gamma_{\xi in} - \gamma_{\xi fin}.$$

DETERMINATION OF THE MATERIAL TEMPERATURE

The temperature field of the material in the course of its motion into the roll space is computed by solving the following equation

$$\left[\frac{\rho c_p W_h}{\sqrt{R_r h_{min}}}\left(1-\frac{3(1+\psi)(\xi^2-\xi_{fin}^2)}{4(1+\xi^2)}(1-\varepsilon^2)-\frac{(1-\psi)}{2}(1-\varepsilon)\right)\right]\frac{\partial T}{\partial \xi}$$
$$=\left(\frac{\lambda}{h_{min}^2(1+\xi^2)^2}\right)\frac{\partial^2 T}{\partial \varepsilon^2}+K\left(\frac{W_h}{2h_{min}}\right)^{n+1}\left[\frac{3(1+\psi)(\xi-\xi_{fin}^2)}{(1+\xi^2)^2}\varepsilon+\frac{1-\psi}{1+\xi^2}\right]^{n+1}$$

in view of the initial (6) and boundary (9) and (10) conditions, and the temperature field of the material in the course of its motion on the roller by solving Eq. (11) in view of the initial (12) and boundary (13) and (14) conditions.

The initial condition of the calculation of temperature field in the roll space is the temperature of the material that is fed into the rolls.

The initial condition of determination of the temperature field in each of the sections of the material movement on the rollers is a finite temperature distribution at the previous section.

Mathematical models that describe the temperature field in the material during its rolling are boundary-value problem with partial differential equations of parabolic type. The solution of such problems should be conducted according to the finite difference method [1, 5].

DETERMINATION OF THE FORCES ACTING ON THE ROLLERS

Forces acting on the rollers during rolling, are the result of the weight of the rollers, as well as the spacer effort, and friction forces that act on the rollers on the part of the material being deformed in the roll space.

Spacers efforts acting on high-speed and low-speed rollers, are [6]

$$F = KLR_r\left(\frac{W_h}{2h_{min}}\right)^n$$
$$\times\int_{\xi_{fin}}^{\xi_{ini}}\int_{\xi_{fin}}^{\xi}\frac{|A|^n\ \mathrm{sgn}(A)-|B|^n\ \mathrm{sgn}(B)}{1+\xi^2}d\xi d\xi = 0.$$

A vector of the spacer effort F is applied to the surface of the roller at a diametrically-section at a point with coordinate ξ_{cF} which is determined by the position of a center of mass of an area S bounded by the axis of the coordinate ξ and the pressure curve $p = f(\xi)$ in the roll space

$$\xi_{cF} = \int_{\xi_{fin}}^{\xi_{ini}}\xi dS \Bigg/ \int_{\xi_{fin}}^{\xi_{ini}} p d\xi,$$

where pressure p is computed by the following equation [6]

$$p = \frac{KW_h^n\sqrt{2R_r h_{min}}}{(2h_{min})^{n+1}} \times \int_{\xi_{fin}}^{\xi_{ini}} \frac{|A|^n \operatorname{sgn}(A) - |B|^n \operatorname{sgn}(B)}{1+\xi^2} d\xi.$$

An angle β_F (rad) between the vector of the spacer effort F and plane that passes through the axes of the rollers is

$$\beta_F = \arcsin\frac{\xi_{cF}\sqrt{2R_r h_{min}}}{R_r}.$$

Then, values of a component of the spacer effort F_x, lying in the plane which passes through the rolls axes, and of component normal to the plane F_y (i.e., these components are directed along the x and y coordinate axes, respectively) are equal to

$$F_x = F\cos\beta_F;$$

$$F_y = F\sin\beta_F = F\frac{\xi_{cF}\sqrt{2R_r h_{min}}}{R_r}.$$

On the part of the material deformed in the roll space in addition to the pressure on the operational surface of the roller also act tangential stresses that result in arising forces applied to the operational surfaces of the roller and creating a moment of resistance to rotation of the latter. The magnitudes of these forces for high-speed and low-speed rollers are determined by the following expressions

$$P_h = KL\sqrt{2R_r h_{min}}\left(\frac{W_h}{2h_{min}}\right)^n \int_{\xi_{fin}}^{\xi_{ini}} |A|^n \operatorname{sgn}(A)\, d\xi;$$

$$P_l = -KL\sqrt{2R_r h_{min}}\left(\frac{W_h}{2h_{min}}\right)^n \int_{\xi_{fin}}^{\xi_{ini}} |B|^n \operatorname{sgn}(B)\, d\xi.$$

A point of application of the resultant force $P_{h(l)}$ to the roll surface in its diametrical section is determined by the position of a center of gravity of the area bounded by the axis of coordinate ξ and the shear stress curve $\tau_{xyh} = f(\xi)$ (or $\tau_{xyl} = f(\xi)$):

$$\tau_{xyl} = K\left(\frac{W_h}{2h_{min}}\right)^n |B|^n \operatorname{sgn}(B).$$

A technique of determination of the coordinate ξ_{cP} of the center of gravity of the area under the distribution curve of the shear stress for the high-speed and low-speed rollers is similar to the technique of determining the coordinate ξ_{cF}. A value of angle β_P (rad) between the vector of the force P and axis x (separately for high-speed and low-speed roller) is computed according to the following equation

$$\beta_P = \arcsin\frac{\xi_{cP}\sqrt{2R_r h_{min}}}{R_r}.$$

Then a value of the component P_x of the force P lying in the plane passing through the roll axes is

$$\beta_P = P\xi_{cP}\frac{\sqrt{2R_r h_{min}}}{R_r},$$

and value of component P_y which is perpendicular to P_x is $P_y = P\cos\beta_P$.

A vector of the total effort $F_{h(l)\Sigma}$ acting on the roll is calculated as the sum of the vectors of gravity of the roller G_r, spacer efforts $F_{h(l)}$, and friction $P_{h(l)}$ (Fig. 4):

$$F_{h(l)\Sigma} = \sqrt{\left(P_{h(l)}\cos\beta_P - G_r - F_{h(l)}\sin\beta_F\right)^2 + \left(P_{h(l)}\sin\beta_P + F_{h(l)}\cos\beta_F\right)^2}.$$

Figure 4 shows that each tangential force to the roller $P_{h(l)}$ is substituted by a statically equivalent set of forces: force $P_{h(l)}$ applied to the roll axis and moment $M_{h(l)} = P_{h(l)}R_r$.

A specific load on the roller is computed by the equation

$$q_{h(l)\Sigma} = \frac{G_{h(l)\Sigma}}{L\delta}.$$

Torque required to deform directly the material and acting on high-speed and low-speed rollers is calculated by the dependencies

$$M_{\text{h}} = KLR_{\text{r}}\sqrt{2R_{\text{r}}h_{\min}} \times \left(\frac{W_{\text{h}}}{2h_{\min}}\right)^{n}\int_{\xi_{\text{fin}}}^{\xi_{\text{in}}}|A|^{n}\,\text{sgn}\,(A)\,d\xi,$$

$$M_{\text{l}} = -KLR_{\text{r}}\sqrt{2R_{\text{r}}h_{\min}} \times \left(\frac{W_{\text{l}}}{2h_{\min}}\right)^{n}\int_{\xi_{\text{fin}}}^{\xi_{\text{in}}}|B|^{n}\,\text{sgn}\,(B)\,d\xi.$$

The torques applied to the rolls on the part of the universal spindles of the roll drive can be determined by the formula

$$M_{\text{h(l)}\Sigma} = M_{\text{h(l)}} + 2M_{\text{b}},$$

where M_{b} is moment of friction in the bearing roller

$$M_{\text{b}} = d_{\text{pin}}(5000cd_{\text{pin}} + 2.55f_0F_{\text{h(l)}\Sigma}),$$

where c, f_0 are coefficients (for radial spherical roller bearings, DIP, widely used in the rolls of roller machines, $c = 0.15$; $f_0 = 0.002$ [6]).

CALCULATION OF THE POWER OF ROLL DRIVE

The power of the group drive of the rollers, the efficiency of which equals to η_{dr}, is determined by the formula

$$N_{\Sigma} = \frac{N_{\text{h}} + N_{\text{l}}}{\eta_{\text{dr}}},$$

where the power expended by the respective rollers in the deformation of the material is computed by the equation

$$N_{\text{h(l)}} = M_{\text{h(l)}\Sigma}\,\omega_{\text{h(l)}}\,M_{\text{h(l)}\Sigma}\frac{W_{\text{h(l)}}}{R_{\text{r}}},$$

The efficiency of the group drive of the rollers is determined from the following dependence $\eta_{\text{dr}} = \eta_1\eta_2 \times \eta_3\eta_4\eta_5^2$, where $\eta_1 = 0.99$ is an efficiency of an electric motor; $\eta_2 = 0.99$, an efficiency of a bush finger joint; $\eta_3 = 0.90$, efficiency of a block reducer; $\eta_4 = 0.99$, an efficiency of gear couplings; $\eta_5 = 0.90$, an efficiency of universal spindle.

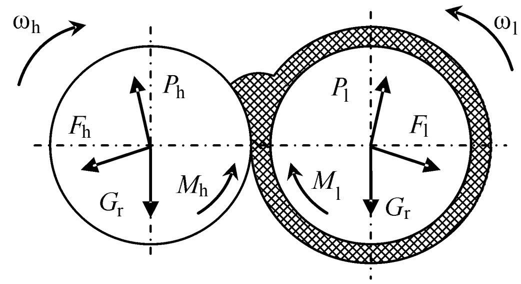

Fig. 4. Scheme of the effort to roller (the material being rolled is on the low-speed roller): M_{h} and M_{l} are torques acting on a high-speed and low-speed bodies of the rollers.

On the basis of the calculation of the total power an electric motor of rollers can be selected.

THERMAL DESIGN OF ROLLERS

Heat and power balance equation of nonisothermal process of rolling is given by

$$Gi_{\text{ini}} \pm Q_{\text{out}} + Q_{\text{diss}} = Gi_{\text{fin}} + Q_{\text{loss}}, \quad (15)$$

where i_{ini} and i_{fin} are a mass enthalpy of the material at the roll inlet and outlet, respectively, J kg^{-1}; Q_{out}, heat energy which is supplied (sign "+") or withdrawn (sign "–") in rolling (external sources of heat setting), W; Q_{diss}, the dissipation energy obtained by material as a result of the irreversible shear strain in the roll space, W; Q_{loss}, heat loss to the environment, W.

Dissipative component of the energy balance of each roll is determined by the formula

$$Q_{\text{diss}} = Q_{\text{diss.h}} + Q_{\text{diss.l}},$$

where $Q_{\text{diss.h(l)}}$ is a dissipation power being provided by the high-speed and low-speed roller:

$$Q_{\text{diss.h}} = \frac{KL\sqrt{2R_{\text{r}}h_{\min}}}{h_{\min}^{n}}\left(\frac{W_{\text{h}}}{2}\right)^{n+1}\left(\frac{1+2n}{n}\right)^{n}\frac{(n+2)}{3^{n+1}} \times \int_{\xi_{\text{fin}}}^{\xi_{\text{in}}}\int_{0}^{1}\left|\frac{3(1+\psi)(\xi^2-\xi_{\text{fin}}^2)}{(1+\xi^2)^2}\varepsilon + \frac{1-\psi}{1+\xi^2}\right|^{n+1}(1+\xi^2)\,d\varepsilon\,d\xi;$$

$$Q_{\text{diss.l}} = \frac{KL\sqrt{2R_{\text{r}}h_{\min}}}{h_{\min}^{n}}\left(\frac{W_{\text{h}}}{2}\right)^{n+1}\left(\frac{1+2n}{n}\right)^{n}\frac{(n+2)}{3^{n+1}} \times \int_{\xi_{\text{fin}}}^{\xi_{\text{in}}}\int_{-1}^{0}\left|-\frac{3(1+\psi)(\xi^2-\xi_{\text{fin}}^2)}{(1+\xi^2)^2}\varepsilon + \frac{1-\psi}{1+\xi^2}\right|^{n+1}(1+\xi^2)\,d\varepsilon\,d\xi.$$

The heat expended on a change in the enthalpy of material in contact with the roller can be calculated by the following equation

$$Q_m = \sum_{j=1}^{k} Gc_p(T_{fin} - T_{ini})_j + \sum_{j=1}^{k} G_{h(l)}c_p(T_{fin} - T_{ini})_j,$$

where T_{fin} and T_{ini} are final and initial temperature of the material passing through the roll space or the section out of the bounds of the roll space that corresponds to jth turnover of the material on the roller ($j = \overline{1, k}$), K; $G_{h(l)}$, productivity provided by the high-speed (low-speed) roller, kg s^{-1} [6]:

$$G_h = \frac{(3 + \psi)}{4} G,$$

$$G_l = \frac{(3\psi + 1)}{4} G.$$

Heat loss of the respective roller is computed by the dependence $Q_{loss} = Q_{loss.f} + Q_{loss.m}$, where $Q_{loss.f}$ and $Q_{loss.m}$ are heat loss from the free surface of the roll and surface covered by the material, respectively, W:

$$Q_{loss.f} = \alpha_f S_f (T_{h(l)} - T_{env}) = \alpha_f L (R_r + \delta)\gamma\xi_{f.h(l)} (T_{h(l)} - T_{env}),$$

$$Q_{loss.m} = \alpha_m S_m (T_{h(l)} - T_{env}) = \alpha_m L R_r \gamma \rho_{m.h(l)} (T_{h(l)} - T_{env}),$$

where S_f and S_m are the area of the free surface of the roller and surface covered by the material, respectively, m^2.

The heat transfer coefficients α_f and α_m (W m^{-2} K^{-1}) are computed by the equations

$$\alpha_f = \alpha_{rad.f} + \alpha_{conv.f},$$
$$\alpha_m = \alpha_{rad.m} + \alpha_{conv.m},$$

where $\alpha_{rad.f}$ and $\alpha_{rad.m}$ are coefficients of heat transfer by radiation

$$\alpha_{rad} = 5.67 \times 10^{-8}\, \varepsilon_{r(m)} \frac{T_{r(m)}^4 - T_{env}^4}{T_{r(m)} - T_{env}},$$

$\varepsilon_{r(m)}$ is emissivity of the surface of the roller (steel ε_r = 0.52...0.56; cast iron ε_r=0.6...0.7) and the material processed (ε_m=0.93...0.95); $T_{r(m)}$, the temperature of the respective roller and material on this roller; $\alpha_{conv.f}$ and $\alpha_{conv.m}$ are convection heat transfer coefficients calculated using criterial equations describing heat transfer from a horizontal rotating cylinder [7].

If the heat exchange occurs under conditions of free convection ($\mathrm{Re} \le \sqrt{\mathrm{Gr\,Pr}}$), the value of the Nusselt number Nu is calculated by the equation

$$\mathrm{Nu} = 0.456(\mathrm{Gr\,Pr})^{0.25}, \quad (16)$$

where Gr and Pr are Grashof and Prandtl criteria.

At Re ≤ 5 × 10^4 heat transfer occurs under the joint influence of the free and forced convection:

$$\mathrm{Nu} = 0.18[(0.5\mathrm{Re}^2 + \mathrm{Gr})\mathrm{Pr}]^{0.315}. \quad (17)$$

At Re > 5 × 10^5 heat transfer occurs under influence of the forced convection:

$$\mathrm{Nu} = \frac{\mathrm{Re\,Pr}\sqrt{0.5f_D}}{5\mathrm{Pr} + 5\ln(3\mathrm{Pr} + 1) + \sqrt{0.5f_D} - 12}, \quad (18)$$

where f_D is coefficient whose value is calculated depending on the value $C = \mathrm{Re}\sqrt{f_D}$: C= –1.828 + 1.77ln C if C ≥ 950; Re/C= –3.68 + 2.04ln C if C < 950.

In Eqs. (16)–(18) we took the following notations:

$$\mathrm{Re} = \frac{2R_r W_{h(l)}}{\nu_{env}}; \quad \mathrm{Nu} = \frac{2\alpha_{env} R_r}{\lambda_{env}}; \quad \mathrm{Gr} = \frac{8g R_r^3}{\nu_{env}^2}\beta_{env}\Delta T;$$

$$\mathrm{Pr}_{env} = 0.7; \quad \beta_{env} = \frac{1}{T_{env}},$$

where ν_{env} and λ_{env} are the kinematic viscosity (m^2 s^{-1}) and thermal conductivity (W m^{-1} K^{-1}) of the environment; β_{env}, temperature coefficient of the environment K^{-1}; g, acceleration due to gravity, m s^{-2}.

Physical properties of the environment are selected at temperature $T = (T_{env} + T_{r(m)})/2$.

After the appropriate calculations based on Eq. (15) we can determine the value of Q_{out}.

In contemporary rollers the heat setting (heating or cooling) of the rollers is provided by a cooling agent, which moves in the peripheral channels made evenly around the circumference along the roll body [1, 3] (Fig. 5).

Cooling agent flow G_{ca} can be computed based on a difference of its temperatures ΔT_{ca} at an inlet and

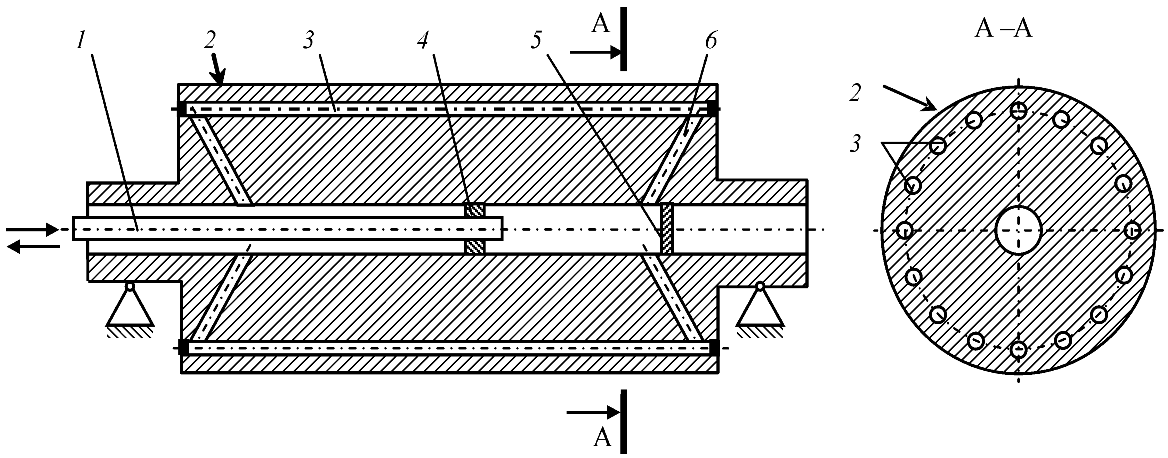

Fig. 5. Scheme of the roll with the peripheral channels: (*1*) pipe for cooling agent supply, (*2*) roll body, (*3*) peripheral channel, (*4*) piston, (*5*) cap, (*6*) sloping channel.

outlet from the channels:

$$G_{ca} = \frac{Q_{out}}{c_{ca}\Delta T_{ca}},$$

where c_{ca} is mass heat capacity of the cooling agent.

An average temperature of the cooling agent preliminary is taken for water $T_{ca} = T_{h(l)} \pm 10$ and for organic and mineral cooling agent, $T_{ca} = T_{h(l)} \pm 20$.

A velocity of the cooling agent in the peripheral channels is calculated by the equation

$$w_{ca} = \frac{G_{ca}}{\rho_{ca} S_{ca}} = \frac{4G_{ca}}{\rho_{ca} n_{ca} \pi d_{ca}^2},$$

where S_{ca} is a total area of a cross-section of the peripheral channels; d_{ca} and n_{ca} are a diameter and amount of the peripheral channels; ρ_{ca} is density of the cooling agent as function of temperature.

The heat transfer coefficient of cooling agent to the channel wall is calculated by the dimensionless equation describing heat transfer in forced flow in channels [1, 6]: if $Re_{ca} > 10^4$, then

$$Nu_{ca} = 0.021 Re_{ca}^{0.8} \; Pr_{ca}^{0.43} \left(\frac{Pr_{ca}}{Pr_w}\right)^{0.25};$$

if $2000 \le Re_{ca} \le 10^4$, then

$$Nu_{ca} = 0.008 Re_{ca}^{0.9} \; Pr_{ca}^{0.43} \left(\frac{Pr_{ca}}{Pr_w}\right)^{0.25};$$

if $Re_{ca} < 2000$, then

$$Nu_{ca} = 015 Re_{ca}^{0.33} \; Gr_{ca}^{0.1} \; Pr_{ca}^{0.43} \left(\frac{Pr_{ca}}{Pr_w}\right)^{0.25},$$

where

$$Gr_{ca} = \frac{g d_{ca}^3}{\nu_{ca}^2} \beta_{ca} \Delta T;$$

$\Delta T = T_{ca} - T_w$ (heating) or $\Delta T = T_w - T_{ca}$ (cooling), where T_w, temperature of the channel wall: $T_w = T_r \pm Q_{out} R_w$, where R_w is thermal resistance of the channel wall [1, 6]:

$$R_w = \frac{\ln\left[\frac{D_{ca}}{n_{ca} d_{ca}} \operatorname{sh}\left(n_{ca} \frac{2R_r - D_{ca}}{D_{ca}}\right)\right]}{2\pi L \lambda_r n_{ca}},$$

where D_{ca} is a diameter of centers of the peripheral channels; λ_r, thermal conductivity of the roll material.

The heat transfer coefficient from the cooling agent to the channel wall

$$\alpha_{ca} = \frac{Nu_{ca} \lambda_{ca}}{d_{ca}}.$$

Then the temperature of the cooling agent is $T_{ca} = T_r \pm Q_{out} R_\Sigma$, where $R_\Sigma = R_w + R_{ca} = R_w + (\pi L \alpha_{ca} d_{ca} n_{ca})^{-1}$.

Thermal calculation is carried out for each a roller. At the same time the average coolant temperature preset in the beginning of the calculation is compared with that calculated. If the preset and calculated temperatures differ by more than 5%, the calculation of the cooling agent temperature is repeated.

When designing the rolls two versions of the parametric and thermal design is advisable to make with a maximum speed of rotation of the rollers, the minimum possible frictions in the roll space, as well as the minimum acceptable initial temperature of the material (in loading on rolls). In the first case, the calculation is conducted with a minimum thickness of the product (the maximum spacer effort occurs), while in the second case, with a maximum (the maximum power that is fed to the rolls). The results of calculations are the starting data to calculate the elements of the rollers on the strength and stiffness and to select the motor of the roller drive.

Initial data for calculation are: a scheme of feeding and removal of the material from the roller according to Fig. 1; type of the roller drive (general, individual); type of the rollers (with the peripheral channels; with a central cavity); radius of the roll body and its weight; minimum size of the roll space; the distance between the restrictive arrows; width of a continuous strip of the material withdrawn from the rollers, a speed of the high-speed and low-speed rollers; the coefficient of frictions in the roll space; temperature of high-speed and low-speed rollers, the real and the initial maximum temperature of the material; type of cooling agent (water, organic high-temperature heat transfer fluid); rheological and thermal properties (density, mass specific heat, thermal conductivity) of the material and cooling agent as a function of temperature; the dimensionless coordinates of the material outlet from the roll space and inlet of the material in the roll space.

Based on the initial data we conducted calculation of the low-speed roll, determined the initial coordinate of the section of the roll space deformation, calculated temperature fields of the material in the roll space and outside it.

If the local temperature of the material exceeds the permissible value then the input data should be changed (usually it is required reduce the angular velocity of the rolls, the roll temperature, or the value of the roll space, and change the friction coefficient).

The power of the roll drive is calculated like the spacer efforts.

After that we determines the dissipation energy in the roll space and parameters of the heat setting of the rolls (temperature and flow of cooling agent in each of the rolls). If one or more parameters of the heat setting of rolls are larger than the permissible value then the initial data should be changed (in this case it is usually necessary to change the angular velocity of the rolls, the value of the roll space, friction coefficient, temperature of the rollers or the type of the cooling agent).

Finally the calculation results are analyzed.

EXPERIMENTAL

The suggested technique of calculation was tested in rolling the composition of the plasticized polyvinyl chloride and also polyolefin compositions with various fillers on the double roller machine of department of

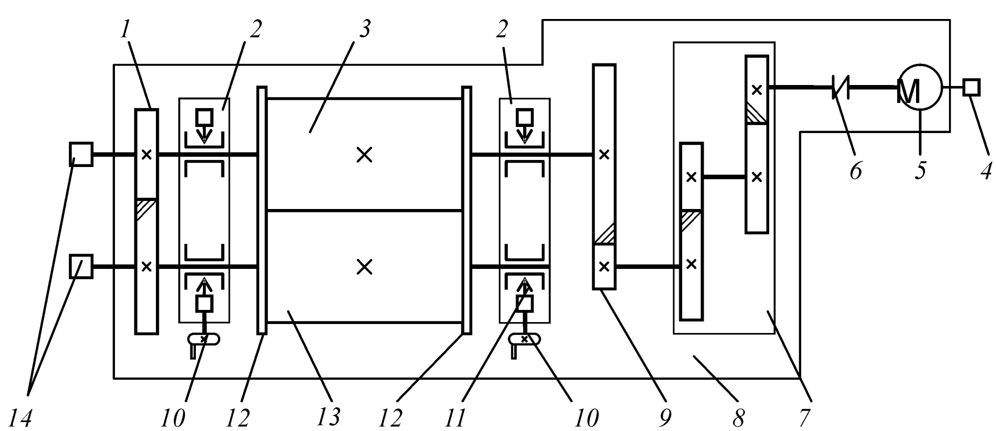

Fig. 6. Scheme of roll setup: (*1*) Friction gears, (*2*) frames, (*3*) rear roll, (*4*) tachometer, (*5*) the drive motor, (*6*) clutch, (*7*) gearbox, (*8*) welded base, (*9*) drive gear, (*10*) mechanisms for moving the front roll, (*11*) load cells, (*12*) restrictive arrows, (*13*) front roll; (*14*) Collectors for supply of electric current to the heaters of roll.

machines and equipment of chemical and oil refineries (NTU "Kyiv Polytechnic Institute," Table 1, Fig. 6) [5].

A roller setup is mounted on a welded base *8* and includes two frame *2* with mechanisms *10* for moving the front roll *13*. Roller bearings were placed in the frames strapped by traverses. Still bodies of roller bearings of a rear roll *3* rest on load cells *11*, which take on the spacer efforts acting on the roll during the roll setup operation. The spacers efforts acting on the front roll are also transmitted to the load cells.

The restrictive arrows *12*, which prevent the leakage of the material over the edges of the roll space were established at the ends of the roll bodies *3* and *13* on the traverses. The operational surfaces of the roll bodies are hardened, ground, and polished.

The roll drive consists of a motor *5*, the shaft of which is connected by means of a clutch *6* with gearbox *7*. The torque from the gearbox is transmitted by means of the drive gear *9* to the rear roller, and then after a couple of friction gears *7*, to the front roll. Testing of a rotor speed of the electric motor is carried out with tachometer *4*. Heating of the rolls occurs by means of induction heaters located in the peripheral channels of the roll bodies. An electric current is supplied to the heaters through a collector *14*.

Governing the temperature of the operational surfaces is ensured by high-speed non-contact systems with proportional control law.

On the necks of the rollers for measuring the torque there are strain gauges whose signals by current collectors are fed to a recording device.

Comparison of experimental and calculated data shows their satisfactory agreement (the discrepancy does not exceed 15%).

CONCLUSIONS

The above method has proven its effectiveness during the design and operation of industrial rollers of different sizes (550 × 800 mm, 550 × 1500 mm, 660 × 2100 mm, 950 × 2400 mm) of the plant "Bolshevik" (Kiev, Ukraine). It allows selection of rational modes of material rolling on the basis of multivariant calculations with using polymers and rubbers.

Technical characteristics of the double roll machine

Parameter	Characteristics
Number of rollers	2
Location of rollers	horisontal
Dimensions of roll body: length × diameter, mm	250 × 200
Heat setting system	
type	Induction
power, kW	8.8
maximum temperature of the roll surface, ° C	350
Value of the roll space, mm	0…10
Peripheral speed of the high-speed roller, m s^{-1}	0.038…0.272
Electric motor of drive:	
power, kW	6
rotor speed, rps	3.3…26.7
Coefficient of frictions in the roll space	1.00; 0.92; 0.84; 0.77; 0.70; 0.65; 0
Overall dimensions (length × width × height), m	1.43 × 0.93 × 1.17

NOTATIONS

A, *B*, *C* are calculation complexes;
b, width of the continuous strip of the material withdrawn from rollers, m;
c, mass heat capacity, J kg^{-1} K^{-1};
d, *D,* diameter, m;
f, friction in the roll space;
g, acceleration of free fall, m s^{-2};
G, mass flow, kg s^{-1};
Gr, Grashof criterion;
h, half of the roll space value, m;
i, mass enthalpy, J kg^{-1}
j, serial number;
K, consistency index, Pa s^{-n};
L, the distance between restrictive arrows of the rolls, m;
M, torque, N m^{-1};
n, exponent of rheological equation;
N, power, W;
Nu, Nusselt number;
p, pressure, Pa;
Pr, Prandtl number;
q, heat flux, W m^{-2};
q_{diss}, intensity of dissipation energy, W m^{-3};
Q, heat flow, W;
r, coordinate directed along the roll radius, m;
R, radius, m;

Re, Reynolds number;
S, area, m^2;
t, time, s;
T, temperature, °C;
w, linear velocity of material, m s^{-1};
W, linear velocity of operational surface of roll, m s^{-1};
x,y,z, Cartesian coordinates;
α, heat transfer coefficient, W m^{-2} K^{-1};
β, temperature coefficient of the rheological equation;
β_{env}, temperature coefficient of the environment, 1 K^{-1};
γ, central angle of the roll section, rad;
δ, thickness of material strip withdrawn from rolls, m;
ε, dimensionless analogue of coordinates y;
η, efficiency;
λ, heat conductivity, W m^{-1} K^{-1};
ν, kinematic viscosity, m s^{-1};
ξ, dimensionless analogue of coordinates x;
ρ, density, kg m^{-3};
τ, shear stress, Pa;
ψ, coefficient of frictions in a roll space;
ω, angular velocity of roll, rad s^{-1}.

indices:

0 is for an initial value;
min, for minimal value;
x,y,z, for corresponding Cartesian coordinates;
h, for high-speed roll;
r, for operational surface of roll;
out, for external heat setting of roll;
in, for inlet of material in the roll space;
diss, for dissipation;
fin, for end of section of deformation in roll space;
conv, for convective heat exchange;
rad, heat transfer by radiation;
m, for material being rolled;
ini, for starting of section of deformation in roll space;
env, for environmental;
b, for roll bearing;
loss, for heat loss;
dr, for roll drive;
f, for free roll surface;
l, for low-speed roll;
ca, for cooling agent;
pin, for roll pin;
c, for center of gravity.

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