ISSN 1070-4272, Russian Journal of Applied Chemistry, 2012, Vol. 85, No. 9, pp. 1475−1481.

Simulation of Disk Extruder Operation

I. O. Mikulionok and L. B. Radchenko†

National Technical University of Ukraine "Kyiv Polytechnic Institute," Kyiv, Ukraine
email: i.mikulionok@kpi.ua

Received December 22, 2011

Abstract—Mathematical simulation of thermoplastic polymer processing by a disk extruder was considered. The results were compared with experimental data obtained in an extruder of a 200 mm diameter. The developed technique of calculation allows selection of design parameters of operational parts and disk speed for preset output of the disk extruder as well as energy-power parameters of extrusion.

DOI: 10.1134/S1070427212090273

An increase in output and diversification of polymers and plastics require designing of efficient devices for their processing. Fairly universal constructions of disk and combined screw-disk extruders were developed together with applying in industry single and twin screw extruders of various sizes and purposes [1–4].

Despite relatively low operating pressure the disk extruders possess advantages in comparison with the screw extruders: high efficiency of mixing, short processing time of material in extruder, simpler construction, and less material and power consumption. Therewith pressure at an outlet of the disk extruder can be improved, e.g., with the aid of a gear pump. Especially effectively the disk extruders can be applied in technological schemes of polymer granulating (also including secondary polymers), and also in a stage of obtaining a melt in cascade schemes in production of filled compositions. These extruders are fairly versatile since allow processing of various polymers and compositions without changes in the construction.

In one of the most effective designs of the disk extruders [2, 3] polymer by a feeder or directly from the hopper (no dosing) goes into the charging inlet of a body and by multiple screw thread made on the cylindrical surface of the rotor, is conveyed to the disk gap formed by motionless body and disk. Pre-melting of polymer occurs in a channel of the disk screw thread, and final melting and homogenization of the melt, in the disk gap. Afterward the prepared melt exits through a central outlet of the body (Fig. 1).

The dosing of the polymer in the extruder caters for effective governing the process of preparation of the melt and its quality at the preset output by changing the disk speed and the size of the disk gap.

Mathematical modeling and experimental investigation of the disk extrusion of thermoplastics

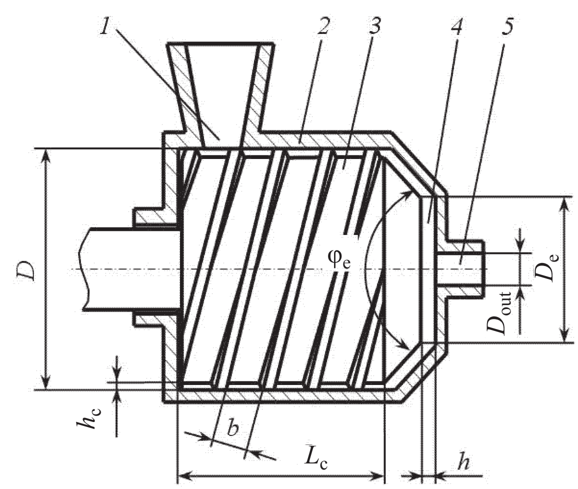

Fig. 1. Scheme of the disk extruder: (*1*) feeding inlet; (*2*) body; (*3*) disk; (*4*) disk gap; (*5*) central outlet of the body; (D, D_e, D_{out}) diameters of cylindrical and end parts of the disk, and also of the central outlet of the body, respectively, m; (L_c, b, h_c) length, width, and depth of the screw thread of the cylindrical part of the disk, m; (h) size of the disk gap, m; (φ_e) angle of taper of the end part of disk, deg.

was considered in detail in [5–8] for the case when the extruder output was controlled by the effect of the normal stresses (Weissenberg effect). In this case, the practical use of the results requires complex rheological relationships taking into account the viscoelastic properties of the melt of material processed. At the same time, in the construction of industrial extruders the screw thread for enforced feeding the polymer to the disk gap is carried out on the side surface of the disk to stabilize the polymer melting. However, an analysis of the disk extrusion shows that an effect of a radial component of melt velocity on the dissipation rate compared with a tangential velocity component is negligible. Thus, a technique of engineering calculation of disk extrusion was required for taking into account the material processing in a screw thread, as well as in the disk gap.

MATHEMATICAL MODELING OF THE DISK EXTRUSION

In the course of the extrusion the polymer sequentially passes through the stages of feeding, melting, and homogenization, which are described by different mathematical models. The above stages occur in the channel of screw thread of a cylindrical surface of the disk, as well as in the disk gap (Fig. 1). An approach suggested in [9–11] to the screw extruding can be used to analyze the polymer processing in the channel of the screw thread of the cylindrical surface of the disk. Since a diameter of disks of industrial extruders is usually at least 160 mm, their screw thread is usually multistart (usually four- or eight-start), and the extruders usually work virtually in adiabatic mode, the polymer processing in the screw thread can be viewed using a plane-parallel model of disk extrusion [9] (Fig. 2). In this case, the processing in the channels of the screw thread is similar to that in a channel of a screw assuming that the performance of each channel is $1/k$ of the performance of the single screw extruder (where k is number of starts the disk thread).

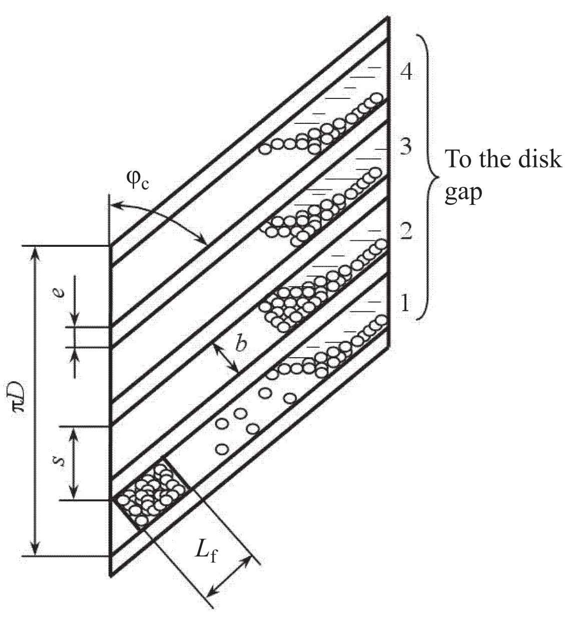

Fig. 2. Scheme of the four start (k = 4) screw thread of the disk deployed on the surface: (s, e, φ_c) pitch (m), crest width (m), and flank angle (deg) of spiral thread of cylindrical disk surface; (L_f) length of feeding inlet of the body, m; (1–4) numbers of thread coils of disk.

Each of the coils of the thread is under the charging inlet of the body for the $1/k$ share of the disk turnaround ($k = \pi D/s$).

Analysis of experimental data shows that the optimal size of the disk gap h is 2.5–3.5 mm [3]. In this case, the resistance of the disk gap remains approximately constant, while with decreasing of the gap less than 2.5 mm it is much higher, and with increasing more than 3.5 mm there are no adequate mixing effect and temperature regime of the processing. The resistance at the outlet of the disk extruder must not exceed 1 MPa, as its increase leads to a significant reduction of the output. This extruder operates in a “hunger” feeding mode.

For the case when the polymer is fed directly to the extruder from the hopper (no dosing), we obtained equations relating the diameter of the disk D, width of its screw thread b, and disk speed n_d (rps) [3]:

$$b = \sqrt{\frac{\pi D G_M \sin\varphi_c}{hkL_f\rho_b}\left(\frac{2h}{g}\right)^{0.25}};$$

$$n_d = \frac{G_M}{bhkL_f\rho_b},$$

where G_M is mass output, kg s^{-1}; ρ_b, bulk density of polymer, kg m^{-3}; g, acceleration of gravity, m s^{-2}.

Thus, in the absence of dosing for the given output of the extruder the specified parameters D, b, and n_d are interrelated and can not be selected arbitrarily. In the case of dosing of polymer the output of the extruder decreases and then the disk parameters must be the same as for the operating case without dosing.

Since a mixture of melt and the solid polymer enters the input to the disk gap then turns up a need of transition region, where completes melting of the residual unmelted polymer (Fig. 3).

The pressure in the disk gap due to an effect of normal stress increases from the periphery of the disk to its center. This leads to the fact that the residuals of unmelted polymer are pushed in the periphery of the disk gap and their melting occurs due to an intense energy dissipation in the melt films of a thickness δ_1 and δ_2 (Fig. 3a) formed near the disk and body.

The process in the disk gap is considered as a sequence of finite annular volumes of a width *dr* selected on the radii *r* ($0 < r < D/2$), and deployed in the plane (Figs. 3a, 3b). This allows to simplify the mathematical model replacing the continuous change in the parameters by their step change [3, 9], and assuming that the values of *dr* is sufficiently small to maintain a required accuracy of the calculations. For further analysis we choose a rectangular coordinate system directing an axis *r* along the radius of the disk; an axis θ, in the tangential direction; and an axis *z*, along a height of the disk gap.

First we distinguish a volume in a region where the melting ends. The peripheral velocity of the disk is much higher than the radial velocity of the polymer, thus, the latter can be neglected in determining the intensity of dissipation. As the distinguished volume is small enough, the heat transfer within it can be assumed axisymmetric, and the convective component in the energy equation can be neglected. Viscous friction forces in the films of the melt and, consequently, their thickness, we assume the same ($\delta_1 = \delta_2 = \delta$). Then, using these assumptions, the energy equation for the films of the melt at the disk and the body has the form

$$\lambda \frac{\partial^2 T}{\partial z^2} + q_V = 0, \quad (1)$$

where *T* is temperature, °C; λ, thermal conductivity of the polymer as a function of temperature, W m^{-1} K^{-1}; q_V, the bulk density of the heat flow of internal energy sources (in the absence of chemical and other interactions this is the intensity of dissipation in the films of the melt), W m^{-3}.

The boundary conditions for Eq. (1) have the form:

$$\left.\frac{\partial T}{\partial z}\right|_{\substack{z=0 \\ z=h}} = 0; \quad (2)$$

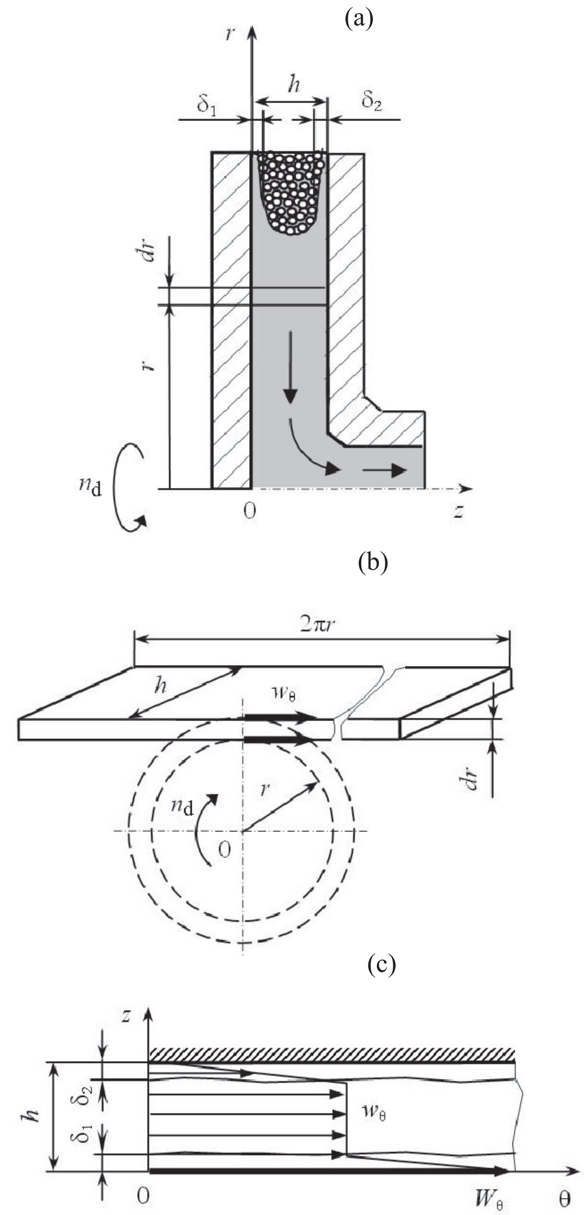

Fig. 3. Scheme of the process in the disk gap: (a) melting of polymer in the disk gap; (b) calculated scheme of the disk gap; (c) melt flow in the disk gap (W_θ is the peripheral velocity of disk, m s^{-1}; w_θ is the peripheral component of polymer velocity in the gap, m s^{-1}; (*r*) radius of location of an elementary annular polymer volume of the width *dr*, m.

$$T\big|_{\substack{z=\delta \\ z=h-\delta}} = T_m, \quad (3)$$

where T_m is the melting point of polymer, °C.

The condition (2) means lacking of the heat transfer in boundary of the disk gap; and condition (3), the fact that the temperature on the boundary of a polymer plug is equal to the melting point of polymer.

The largest component of the stress tensor in the disk gap is $\tau_{z\theta}$, which depends on the tangential component w_θ of velocity. Since the thickness of the melt film is small, then it can be assumed with sufficient accuracy for engineering calculations that the rate of the melt in the films w_θ changes linearly, and its relative value varies in the range [0; $W_\theta/2$] (Fig. 3c). The melt viscosity within the selected volume is assumed constant and is calculated from the rheological equation as a function of shear rate $\dot{\gamma}_\theta$

$$\dot{\gamma}_\theta = \frac{\partial w_\theta}{\partial z} = \frac{W_\theta}{2\delta} \tag{4}$$

and average temperature in this volume

$$\overline{T} = \frac{1}{\delta}\int_0^\delta T dz. \tag{5}$$

Then the intensity of dissipation is

$$q_V = \tau_{z\theta}\dot{\gamma}_\theta = \mu\dot{\gamma}_\theta^2 = \mu\left(\frac{W_\theta}{2\delta}\right)^2, \tag{6}$$

where μ is a dynamic viscosity of the melt, Pa s.

Let us determine the thickness of the melt film and consider the heat balance equation for a melting surface of the solid polymer from disk

$$2\pi r\, dr q_{|z=\delta} = dG_{\mathrm{Mm}}(i_\mathrm{m} - i_\mathrm{in}), \tag{7}$$

where

$$q_{|z=\delta} = -\lambda \left.\frac{\partial T}{\partial z}\right|_{z=\delta} = q_V\delta = \frac{\mu W_\theta^2}{4\delta}$$

is the surface of heat flow to the unmelted polymer, W m^{-2}; dG_{Mm}, mass flow of the formed melt, kg s^{-1}; i_m and i_in are mass enthalpy of the polymer at its melting point T_m and at temperature T_in of granules entering the extruder, respectively, J kg^{-1}.

On the other hand, under the steady-state conditions the melt within the selected volume is distributed by the surface of the rotating disk with an average peripheral velocity $W_\theta/2$ (Fig. 3c), and, therefore for the mass flow of the melt we can write

$$G_{\mathrm{Mm}} = \frac{1}{2}\rho\delta W_\theta\, dr, \tag{8}$$

where ρ is the polymer density as function of the temperature, kg m^{-3}.

Substituting Eq. (8) in relation (7), we obtain

$$\delta = \frac{4\pi r q_{|z=\delta}}{\rho W_\theta (i_\mathrm{m} - i_\mathrm{in})}. \tag{9}$$

The set of Eqs. (1)–(9) gives relation for calculating:

– a temperature field in the melt film

$$T = T_\mathrm{m} + \frac{\mu W_\theta^2}{8\lambda}\left(1 - \frac{z^2}{\delta^2}\right);$$

– the average temperature of the melt film

$$\overline{T} = T_\mathrm{m} + \frac{\mu W_\theta^2}{12\lambda};$$

– the average thickness of the melt film

$$\delta = \sqrt{\frac{\pi r \mu W_\theta}{\rho(i_\mathrm{m} - i_\mathrm{in})}};$$

– the power of dissipation on the melt film

$$dN = \frac{\pi\mu W_\theta^2}{\delta} r\, dr.$$

For the region of the solid polymer ($\delta \le z \le h - \delta$) and in the region of the melt the energy transfer is described by the equation

$$\rho c w_r \frac{\partial T}{\partial r} = \lambda\frac{\partial^2 T}{\partial z^2} + q_V, \tag{10}$$

where c is a mass heat capacity of polymer as a function of temperature, J kg^{-1} K^{-1}; w_r is the radial component of the polymer velocity, m s^{-1}.

Therewith in the region of the solid polymer $q_V = 0$ [10].

The boundary conditions for solving Eq. (10) have the form:

– for the transition region

$$T|_{r=D/2} = T_0, \tag{11}$$

and also the condition (3);
– for the melt region: the condition (2).

The approximate value of the radial component w_r of the velocity in the melt zone can be calculated from the mass flow

$$w_r = \frac{G_M}{2\pi r h \rho}, \qquad (12)$$

then the dissipation intensity in this region is

$$q_V = \mu \left(\frac{W_\theta}{h} \right)^2. \qquad (13)$$

Assuming that in the direction θ the melt flow occurs solely due to the disk rotation then an equation of motion is written as

$$\frac{\partial \tau_{z\theta}}{\partial z} = 0, \qquad (14)$$

therefore, $\tau_{z\theta} = \tau_0 = \text{const}$.

The corresponding boundary conditions:

$$w_\theta|_{z=0} = W_\theta; \qquad (15)$$

$$w_\theta|_{z=h} = 0. \qquad (16)$$

Rheological equation in general form can be written as

$$\tau_{z\theta} = \mu(I_2, T)\ \dot{\gamma}_\theta, \qquad (17)$$

where I_2 is the second invariant of a strain rate tensor.

Since the melt flow is non-isothermal and rheological equation can be approximated by various functions, the set of hydrodynamic equations (14)–(16) for each element Δr was solved by a numerical method according to the following scheme. The initial value of shear stress τ_0 was set and from Eq. (17) the strain rate and, consequently, the velocity at each node along the coordinate z were found starting from the boundary condition (15). After that, the condition (16) was checked, and if it was not satisfied with the required accuracy, then the refined approximation of value τ_0 was assumed.

The set of Eqs. (10)–(13) is solved by the sweep in view of the fact that physical properties of the material being processed depend on the temperature.

EXPERIMENTAL

For experimental studies was used a disk extruder ED-2 (disk extruder, model 2) developed by members of the National Technical University of Ukraine “Kyiv Polytechnic Institute,” as well as the Public Joint Stock Company “Scientific and Production Enterprise Bolshevik,” and Open Joint Stock Company Ukrainian Research Institute Plastmash (Kiev).

The main design parameters of the extruder ED-2 (Figs. 1, 2): D = 200 mm; D_b = 160 mm; D_{out} = 30 mm;

Fig. 4. Photograph of the disk removed from the extruder (the unmelted polymer is marked by lifgt color).

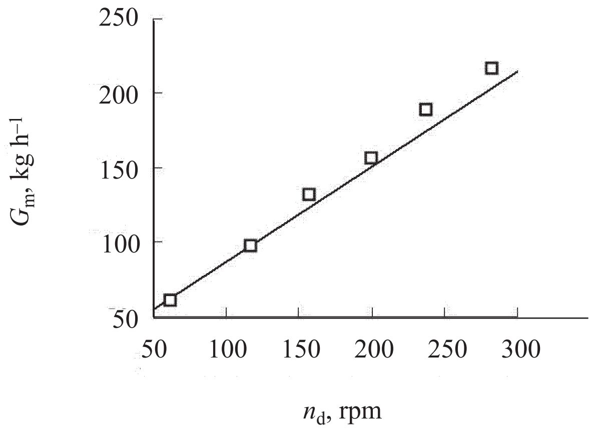

Fig. 5. Output on the disk extruder vs. the disk speed.

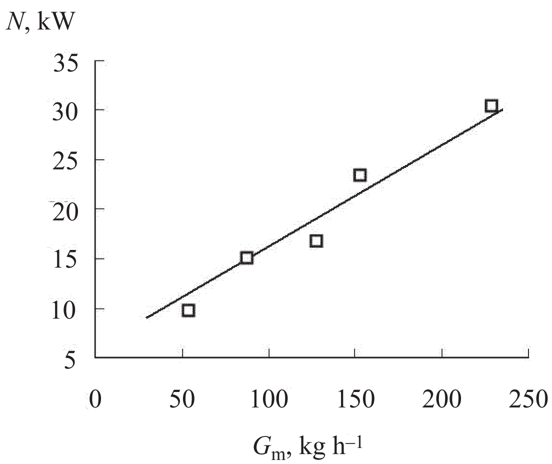

Fig. 6. The drive power of the disk extruder vs. its output.

$L_c = 200$ m; $L_f = 50$ mm; $\varphi_e = 120°$; $h_c = 6$ mm; $k = 4$ (8); $b = 22$ (18) mm.

In the course experiments we varied the disk speed, the size of the disk gap, output, and processing temperature. Experiments exhibited that the operating modes, in which 50–60% of the original polymer in the screw thread is melted, are the most stable. A photograph of the disk removed from the extruder after its stopping during the processing is shown in Fig. 4.

Figures 5 and 6 show dependences of the output on the disk speed and the power on the extruder output in processing LDPE of 15803-020 brand (State Standard GOST 16337-77) (lines denote calculated values; dots, experimental data).

The theoretical results are compared with experimental data obtained in the processing of various materials: polyethylene high and low pressure, polypropylene and polystyrene, including those filled by glass fiber, abrasive particles, dyes, etc. The difference of the experimental and calculated data is not greater than 17%.

CONCLUSIONS

The mathematical model and technique of calculation of a disk extruder for a preset output and mean melt temperature at the outlet of the extruder allow determining the basic parameters of the equipment and processing (diameter and disk speed, minimum width of a screw thread, the minimum required drive power, thermal heterogeneity of melt).

The above method of calculation can be with sufficient accuracy for engineering calculations applied to designing a new or upgrade existing extrusion equipment for processing of thermoplastic polymers and plastics.

DESIGNATION

b is the width of the thread of the cylindrical surface of the disk, m;
c is mass heat capacity, J kg^{-1} K^{-1};
D is the disk diameter, m;
e is the width of the threat crest of the cylindrical surface if the disk, m;
g is acceleration of the free falling, m s^{-2};
G_M is mass output, kg s^{-1};
h is size of channel gap, m;
i is mass enthalpy, J kg^{-1};
k is a number of the threat starts of the cylindrical disk surface;
L is length, m;
n_d is the disk speed, rps;
N is the power, W;
q is surface density of the heat flow, W m^{-2};
q_V the volume density of the heat flow, W m^{-3};
r, θ, z are cylindrical coordinates;
s is pitch of the spiral thread of the cylindrical disk surface, m;
T is temperature, °C;
w, W are linear velocity, m s^{-1};
δ is the thickness, m;
$\dot{g}$ is the shear rate, s^{-1};
λ is thermal conductivity, W m^{-1} K^{-1};
μ is dynamic viscosity, Pa s;

ρ is density, kg m^{-3};
φ_c is the flank angle of the screw thread of the cylindrical disk surface, deg;
φ_e is the taper of the end part of the disk, deg.

The main indices. 0 for the initial value; *M* for the mass; *r*, θ, *z* for the cylindrical coordinate; *V* for the volume; in for the input of the polymer in extruder; d for the disk; f for the feeding inlet of extruder; b for the bulk volume of polymer; m for melting the polymer; melt for the melt; e for the end disk surface; out for central outlet of the body; c for cylindrical part of the disk.

REFERENCES

1. Schenkel, G., *Schneckenpressen fur Kunststoffe*, Carl Hanser Verlag Monchen, 1959.
2. *Diskovye ekstrudery* (Disk Extruders), Ostapchuk, Yu.G., Yazlovitskii, M.L., Bachinskii, K.K., et al., Kiev: Tekhnika, 1972.
3. Radchenko, L.B., *Pererabotka termoplastov metodom ekstruzii* (Processing of Thermoplastics by Extrusion), Kiev: 1999.
4. Rauwendaal, C., *Polymer Extrusion,* Hanser Publications; 2001
5. Kocherov, V.L., Lukach, Yu. E., and Sporyagin, E. A., *Khim. Mashinostr.*, 1972, no. 16, pp. 41–45.
6. Ostapchuk, Yu.G., Dubovitskii, V.F., and Gonchar L.L., *Polimernoe oborudovanie i pererabotka plastmass: Nauchn.-tekhn. sb.* (Polymer Equipment and Processing of the Plasics), Kiev: Tekhnika, 1974.
7. Zhuk, N.K., Kocherov, V.L., and Sporyagin, E.A., , *Khim. Mashinostr.*, 1975, no. 33, pp. 62–71.
8. Sporyagin, E.A., *Dynamics of the Processing of Thermoplastics and Composite Disc Extrusion*, *Doctoral Sci. Dissertation*, Moscow, 1982.
9. Mikulionok, I. O. and Radchenko, L.B., *Khim. Prom-st’,* 2011, vol. 88, no. 6, pp. 281–296.
10. Mikulionok, I. O. and Radchenko, L.B., *Khim. Prom-st’,* 2011, vol. 88, no. 7, pp. 325–334.
11. Mikulionok, I. O. and Radchenko, L.B., *Khim. Prom-st’,* 2011, vol. 88, no. 7, pp. 335–344.