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PROCESSES AND DEVICES  
OF CHEMICAL MANUFACTURES

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## Modeling of the Heat Processing of Continuously Molded Product

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**Abstract**—The mathematical models of heat processing (heating and cooling) of materials, semi-products and products, including multilayer produced by continuous molding, in particular, by extrusion and calendaring are considered. The proposed models allows selection of main sizes of devices of the heat processing line for a preset output or determination of a maximum capacity of the line for given sizes of these devices.

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Sheet, roll, film, tube, rod, and lengthy specialized materials and products with use of polymers and elastomers, including filled and layered with both isotropic and anisotropic structure are widely used as the construction and decoration materials in the building, general and transport engineering, furniture, electrical, chemical and other industries along with traditional materials such as metals, wood, glass, fabric, and paper [1–3].

Processing of these materials is carried out in view of a set of different technological processes ensuring products with specific properties, one of which is heat processing and, in particular, cooling the material from the forming temperature to the temperature in the area of a receiver. The length of the zone of the heat processing can be up to a hundred meters [4], which witnesses the importance of this process in the quality of the product.

The unknown parameter in the calculation of the device for the heat processing is the length of the device (if the technological line output and final temperature of the processed article are known), or the final temperature of the processed article and/or mode of heat processing ensuring this temperature (assuming a given length).

Existing mathematical models and corresponding calculation methods are suitable for the analysis of the heat processing of homogeneous materials [5–7] or multilayer thin materials-coatings of solids [8]. Present-

day continuous and long materials can be of large and complex structure: sheets are made up to 100 mm [9] (the number of layers of sheets and films often reaches seven–nine or more [10]), and pipes are of 3050 mm in external diameter and with the number of layers of four or more [11]. In the course of the analysis of the heat processing of such materials (primarily multilayer, when the individual layers have different thicknesses and properties) these models can lead to significant deviations from the real process.

The purpose of this paper is to develop mathematical models and universal technique of calculation of heat processing (cooling and heating) produced by the continuous molding of homo- and heterogeneous materials and products in the most popular heat processing devices in the industry.

### MATHEMATICAL MODELING OF THE HEAT PROCESSING

Devices, where the heat processing is implemented by convective and radiant heat transfer as well as their combination, is the most widely used by way of impact on the processed article while cooling devices, by the nature of the impact (Fig. 1). Therewith the way of organization of the heat flow in these devices is usually governed by shape, size, and properties of the processed article (Fig. 2).

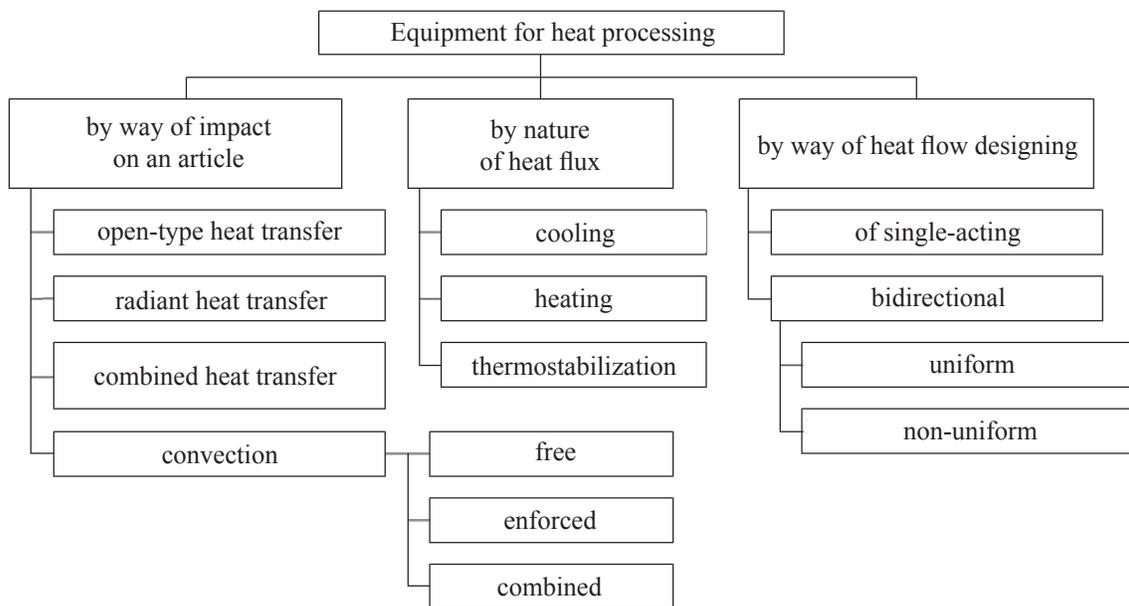


Fig. 1. The classification scheme of equipment for heat processing.

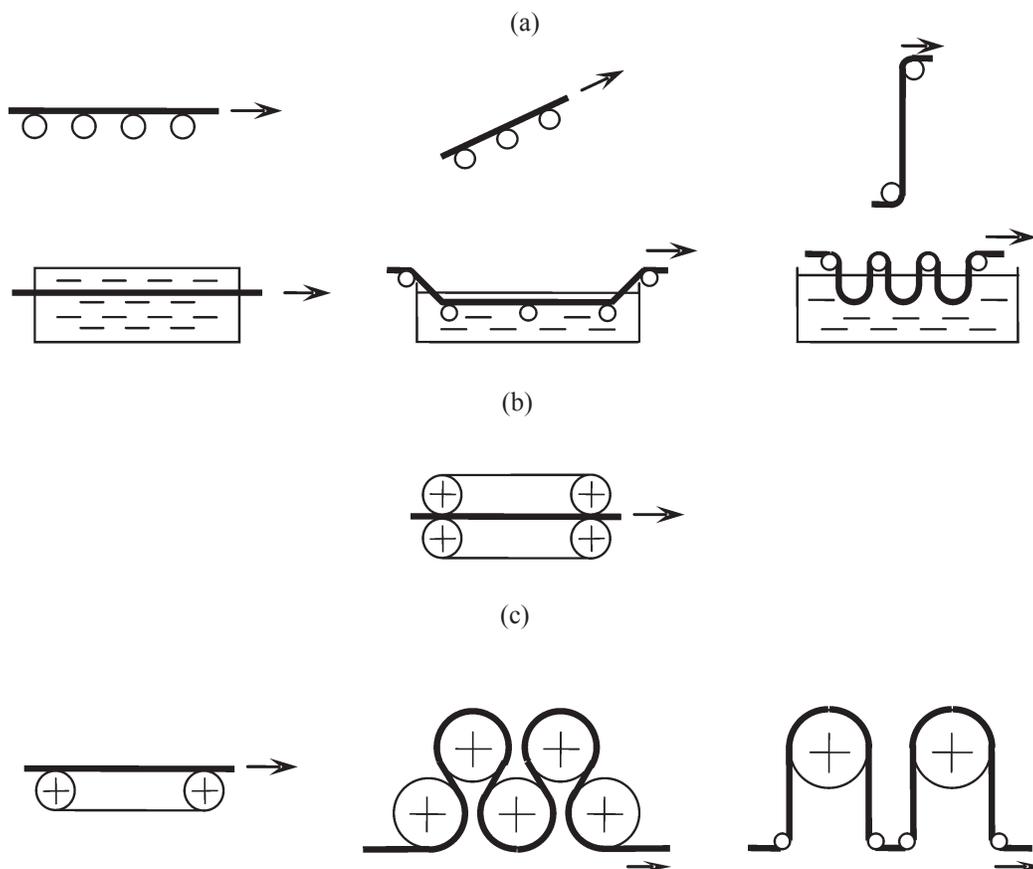


Fig. 2. Main types of devices for heat processing of continuous and long articles: (a) convective (roller conveyor, conveyor, baths of various types), (b) contact, (c) combined (band and drum).

In general case the heat processing is described by the energy equation [1] together with uniqueness conditions.

$$\rho c_p \frac{dT}{dt} = -\nabla q + q_V, \quad (1)$$

where  $\rho$  is density,  $\text{kg m}^{-3}$  as function of temperature  $T$ ,  $^{\circ}\text{C}$ ;  $c_p$ , mass isobaric heat capacity,  $\text{J kg}^{-1} \text{K}^{-1}$  as function of temperature;  $t$ , time, s;  $q$ , surface density of heat flow,  $\text{W m}^{-2}$ ;  $q_V$ , volume density of heat flow of internal sources of energy,  $\text{W m}^{-3}$ .

Since the shape and sizes of a cross section of the articles in course of the heat processing in the most events are almost constant a dissipative component  $q_V$  in Eq. (1) can be ignored. Moreover, only one of the velocity components (velocity of the article motion  $W_z$ ) can be assumed non-zero. Thereby this component in the most cases is constant for all points of the cross-section of the article. Thus, the energy equation in the Cartesian coordinate system has the form

$$\rho c_p \left( \frac{\partial T}{\partial t} + W_z \frac{\partial T}{\partial z} \right) = \nabla (\lambda \nabla T), \quad (2)$$

where  $W$  is a linear velocity,  $\text{m s}^{-1}$ ;  $z$ , coordinate directed along the article motion, m;  $\lambda$ , thermal conductivity,  $\text{W m}^{-1} \text{K}^{-1}$ , as function of temperature.

Since the main processes of molding the products, extrusion and calendaring, are continuous processes for the fixed coordinate system, equation (2) takes the form

$$\rho c_p W_z \frac{\partial T}{\partial z} = \nabla (\lambda \nabla T), \quad (3)$$

and for the moving coordinate system, i.e., related to the processed article accounting for that  $W_z = z/t$ :

$$\rho c_p \frac{\partial T}{\partial t} = \nabla (\lambda \nabla T). \quad (4)$$

Mathematical models describing the distribution of the temperature field in the operational bodies of many types of equipment for processing of thermoplastic materials (including devices for heat treatment) are the boundary problems with linear partial differential equations of second order (usually parabolic).

The equation describing the temperature field in the devices for processing thermoplastic materials can be reduced to the general form [5]:

$$C(T, x, y) \frac{\partial T}{\partial x} - \frac{\partial}{\partial y} \left[ k(T, x, y) \frac{\partial T}{\partial y} \right] - A(T, x, y) \frac{\partial T}{\partial y} = f(T, x, y) \quad (5)$$

in the considering region ( $x_{\text{in}} \leq x \leq x_{\text{f}}$ ,  $y_{\text{in}} \leq y \leq y_{\text{f}}$ ) with the boundary conditions

$$T(t_{\text{in}}, y) = \varphi(y), \quad (6)$$

$$\alpha_1(x) T(x, y_{\text{in}}) + \beta_1(x) \frac{\partial T(x, y_{\text{in}})}{\partial y} = \varphi_1(x), \quad (7)$$

$$\alpha_2(x) T(x, y_{\text{f}}) + \beta_2(x) \frac{\partial T(x, y_{\text{f}})}{\partial y} = \varphi_2(x), \quad (8)$$

where  $x$  and  $y$  are arguments of Eq. (5); indices “in” and “f” denote initial and final values of a physical parameter.

Based on the process conditions coefficients  $C(T, x, y)$ ,  $k(T, x, y)$ ,  $A(T, x, y)$ , and  $f(T, x, y)$  of Eq. (5) take the various values. Coefficients  $\alpha_1$ ,  $\beta_1$ ,  $\varphi_1$  and  $\alpha_2$ ,  $\beta_2$ ,  $\varphi_2$  of the boundary conditions (7) and (8) also depend on conditions in the correspondent equipment or its distinct part.

A starting temperature distribution is determined by temperature distribution over the article cross-section at release from previous unit of equipment or previous part of the equipment [initial condition (6)].

Equation (5) and its solving in the general form together with the boundary conditions (6)–(8) allow developing an unified compute model for numerical research of the temperature field of the article in various types of equipment at the correspondent conditions of carrying out the heat processing.

Equations (3) and (4) are as a rule solved at the boundary conditions of the first or third kind using both similarity and numerical methods. Last for the heat processing of thermoplastic materials are preferred because they allow accounting for the dependence of thermal properties and heat transfer coefficients on the temperature. A method of finite differences (grid method) using the implicit scheme is the most common method of solving such equations [12].

Calculations of the temperature field in the cross-section of the product at each point of the device for heat processing is possible by solving a mathematical model of the heat processing consisting of a heat differential

equation, initial and boundary conditions determined by the design of the device and process parameters. In general, the heat processing occurs in a wide temperature range, therefore, the dependence of the thermophysical properties of materials of a separate structural components as well as material of the article in a whole on the temperature should be taken into account [13]. And, in addition to those mentioned above assumptions, it was assumed that in the point of contact of two adjacent layers of the multilayer material boundary conditions of the fourth kind occur.

Mathematical model of heat processing (further for simplifying mathematical description the cooling as the most widely used process among the heat treatment processes is considered) of sheet, roll, and film material placed on a plane (horizontal, vertical, sloped) should be viewed in Cartesian coordinate system (Fig. 3). Therewith the heat processing of article in a conveyor belt (Fig. 2c) can also be considered as the heat processing of the material in the device of convective type, assuming that one or both outer layers of article correspond to the conveyor belts.

Since a wide of these articles by one or two orders of magnitude higher than their thickness the transient heat conduction equation (4) for the *i*th layer of the layered article in this case will be (for processing of a monolayer article  $\rho_1 = \rho_{tpm}$ ;  $c_1 = c_{tpm}$ ;  $\lambda_1 = \lambda_{tpm}$ )

$$\rho_i c_i \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_i \frac{\partial T}{\partial x} \right) \tag{9}$$

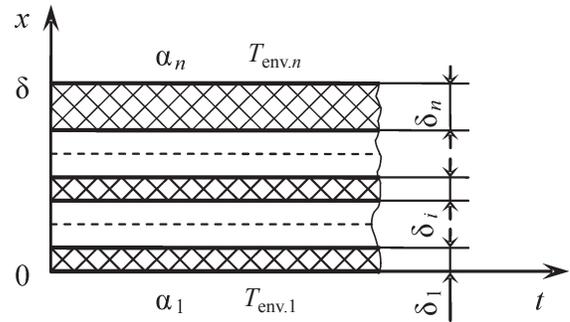
Initial condition:

$$T|_{t=0} = T(x).$$

Boundary conditions:

$$\left. \begin{aligned} \alpha_1(T - T_{env.1}) - \lambda_1 \frac{\partial T}{\partial x} \Big|_{x=0} &= 0, \\ \alpha_n(T - T_{env.n}) + \lambda_n \frac{\partial T}{\partial x} \Big|_{x=\delta} &= 0, \\ T_i \Big|_{x=\sum_i \delta_i} &= T_{i+1} \Big|_{x=\sum_i \delta_i}, \\ \lambda_i \frac{\partial T}{\partial x} \Big|_{x=\sum_i \delta_i} &= \lambda_{i+1} \frac{\partial T}{\partial x} \Big|_{x=\sum_i \delta_i} \end{aligned} \right\} \tag{10}$$

Equation (9) is a quasi-linear parabolic equation of



**Fig. 3.** Scheme of heat processing of the sheet article in the convective device: (*x*) coordinate directed across the article;  $\delta_1, \delta_2, \delta_n, \delta$  is the thickness of *i*th layer of the article ( $i = \overline{1, n}$ ) and of the article as a whole; ( $\alpha$ ) heat transfer coefficient,  $W m^{-2} K^{-1}$ ; ( $T_{env}$ ) environmental temperature, °C.

the general form (5) with boundary conditions (6)–(8).

Reducing Eq. (9) and boundary conditions (10) to the general form we get coefficients of derivatives and right side of Eq. (5):

$$C(T, t, x) = \rho_i c_i; k(T, t, x) = \lambda_i; A(T, t, x) = 0; f(T, t, x) = 0.$$

If the coefficients of the boundary conditions (7) and (8) of Eq. (5) are assumed equal

$$\begin{aligned} \alpha_1(t) = \alpha_1, \beta_1(t) = -\lambda_1, \varphi_1(t) &= \alpha_1 T_{env.1}; \\ \alpha_2(t) = \alpha_n, \beta_2(t) = -\lambda_n, \varphi_2(t) &= \alpha_n T_{env.n}; \end{aligned}$$

then these boundary conditions are appropriate for the type of boundary conditions (10) of Eq. (9).

In the case of cooling of both surfaces of isotropic material with similar intensity ( $\alpha_1 = \alpha_n = \alpha$  and  $T_{env.1} = T_{env.n} = T_{env}$ ) the problem can be considered as symmetrical (Fig. 4) and boundary conditions will be of the following form:

$$\alpha_1(T - T_e) - \lambda_1 \frac{\partial T}{\partial x} \Big|_{x=-\delta/2} = 0 \tag{11}$$

or

$$\alpha_n(T - T_{env}) + \lambda_n \frac{\partial T}{\partial x} \Big|_{x=\delta/2} = 0 \tag{12}$$

$$\frac{\partial T}{\partial x} \Big|_{x=0} = 0 \tag{13}$$

Then the coefficients of boundary conditions (7) and (8) of Eq. (5) have the form:

– for conditions (11) and (13):

$$\alpha_1(t) = \alpha, \beta_1(t) = -\lambda_1, \varphi_1(t) = \alpha T_e;$$

$$\alpha_2(t) = 0, \beta_2(t) = 1, \varphi_2(t) = 0;$$

– for conditions (12) and (13):

$$\alpha_1(t) = \alpha, \beta_1(t) = \lambda_n, \varphi_1(t) = \alpha T_e;$$

$$\alpha_2(t) = 0, \beta_2(t) = 1, \varphi_2(t) = 0;$$

Mathematical model of the cooling of the sheet and film material in the device of drum type should be viewed in the cylindrical coordinate system (Fig. 5).

In this case, the transient heat conduction equation looks like

$$\rho_i c_i \frac{\partial T}{\partial t} = \frac{\partial}{\partial r} \left( \lambda_i \frac{\partial T}{\partial r} \right) + \frac{\lambda_i}{r} \frac{\partial T}{\partial r}, \quad (14)$$

where  $r$  is the current radius, m.

Initial condition

$$T|_{t=0} = T(r). \quad (15)$$

Provided an assured and stable supply of temperature on the surface of the drum the following boundary conditions are set: the conditions of the first kind occur on the article surface, which is in contact with the drum, and the conditions of the third kind occur on the free article surface:

$$T|_{r=R_d} = T_d; \quad (16)$$

$$\alpha_n(T - T_{env}) + \lambda_n \frac{\partial T}{\partial r} \Big|_{r=R_d + \sum_i \delta_i} = 0; \quad (17)$$

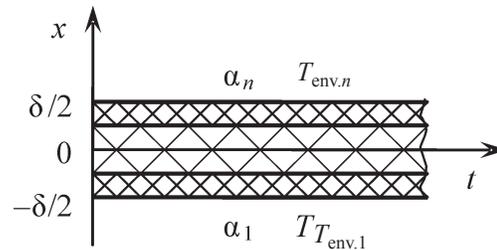
$$T_i \Big|_{x=\sum_i \delta_i} = T_{i+1} \Big|_{x=\sum_i \delta_i};$$

$$\lambda_i \frac{\partial T}{\partial r} \Big|_{r=R_d + \sum_i \delta_i} = \lambda_{i+1} \frac{\partial T}{\partial r} \Big|_{r=R_d + \sum_i \delta_i}. \quad (18)$$

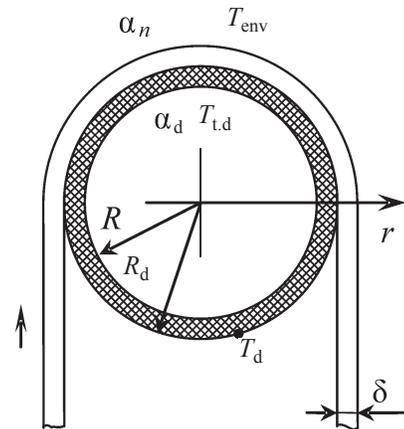
Reducing Eq. (14) to the general form coefficients at the derivatives and right side of Eq. (5) can be obtained:

$$C(T, t, r) = \rho_i c_i; k(T, t, r) = \lambda_i; A(T, t, r) = \lambda_i / r; f(T, t, r) = 0.$$

In view of Eqs. (16) and (17) coefficients of the



**Fig. 4.** Scheme of the symmetric cooling of the sheet isotropic material in the convective device ( $\alpha_1 = \alpha_n = \alpha$  and  $T_{env,1} = T_{env}$ ,  $n = T_{env}$ ).



**Fig. 5.** Scheme of heat processing of sheet, roll, and film article in the device of the drum type: ( $R$  and  $R_d$ ) inner and outer radii of the drum, m; ( $T_{i,d}$ ) temperature of heat transfer agent in the drum, °C; ( $T_d$ ) temperature of the outer surface of the drum, °C; ( $\alpha_d$ ) heat transfer coefficient from the inner surface of the drum shell to the liquid,  $W m^{-2} K^{-1}$ .

boundary conditions (7) and (8) of Eq. (5) look like:

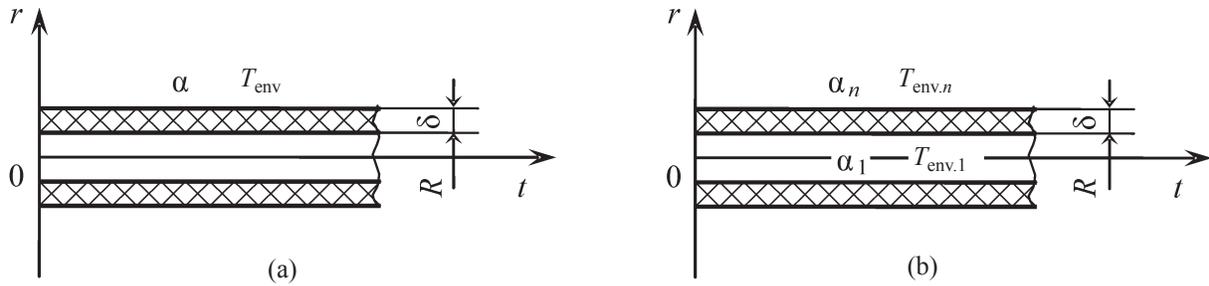
$$\alpha_1(t) = 1, \beta_1(t) = 0, \varphi_1(t) = T_d;$$

$$\alpha_2(t) = \alpha_n, \beta_2(t) = \lambda_n, \varphi_2(t) = \alpha_n T_{env}.$$

The model considered of processing material in the device of the drum type assumes that on the surface of the drum with a significant flow of a heat transfer agent there is a reliable thermal stabilization system of temperature. To select the most efficient heat transfer agent and its parameters, which ensure the required temperature on the surface of the drum, the problem of cooling the material should be solved taking into account the parameters of the heat transfer agent (Fig. 5).

In this case the following boundary conditions are set:

– on the inner surface shell of the drum and free surface



**Fig. 6.** Scheme of heat processing of pipe article in the convective device in the case of (a) mono- (a) and (b) bilateral processing: ( $r$ ,  $R$ ) the current radius and radius of the inner surface of the pipe article,  $m$ .

of the article the conditions of the third kind occur;  
 – on the surface of the article, which is in contact with the drum the conditions of the fourth kind occur;

In Eq. (14) the first layer ( $i = 1$ ) corresponds to the shell of the drum ( $\delta_1 = R_d - R$ ), and residual layers, to the treated article.

Thus, to the boundary conditions (17) and (18) the following conditions should be added

$$\alpha_d(T - T_{t,d}) - \lambda_d \frac{\partial T}{\partial r} \Big|_{r=R} = 0; \quad (19)$$

$$\begin{aligned} T_d &= T_1; \\ \lambda_d \frac{\partial T}{\partial r} \Big|_{r=R} &= \lambda_1 \frac{\partial T}{\partial r} \Big|_{r=R}. \end{aligned}$$

Moreover, the initial condition relative to a thickness of the drum shell is added to an initial condition relative to the article thickness

$$T|_{t=0} = T_{d(r)}.$$

In solving Eq. (14) for the drum shell the coefficients at derivatives and right side of Eq. (5) take the form:

$$C(T, t, r) = \rho_d c_d; \quad k(T, t, r) = \lambda_d; \quad A(T, t, r) = \lambda_d / r; \quad f(T, t, r) = 0.$$

Accounting for Eqs. (17) and (19) coefficients of the boundary conditions (7) and (8) of Eq. (5) look like:

$$\begin{aligned} \alpha_1(t) &= \alpha_d, \quad \beta_1(t) = -\lambda_d, \quad \varphi_1(t) = \alpha_d T_{t,d}; \\ \alpha_2(t) &= \alpha_n, \quad \beta_2(t) = \lambda_n, \quad \varphi_2(t) = \alpha_n T_{env}. \end{aligned}$$

Mathematical model of the cooling of a tubular material should be also viewed in the cylindrical coordinate system (Fig. 6).

In the case of the traditional single sided cooling of a pipe (Fig. 6a) the transient heat conduction equation has the form of (14), the initial condition, of (15), and the boundary conditions:

$$\left. \begin{aligned} \frac{\partial T}{\partial r} \Big|_{r=R} &= 0; \\ \alpha(T - T_{env}) + \lambda_n \frac{\partial T}{\partial r} \Big|_{r=R+\delta} &= 0, \\ T_i \Big|_{x=\sum_i \delta_i} &= T_{i+1} \Big|_{x=\sum_i \delta_i}, \\ \lambda_i \frac{\partial T}{\partial r} \Big|_{r=R+\sum_i \delta_i} &= \lambda_{i+1} \frac{\partial T}{\partial r} \Big|_{r=R+\sum_i \delta_i} \end{aligned} \right\} \quad (20)$$

Then accounting for (20) and (21) the coefficients of the boundary conditions (7) and (8) of Eq. (5) look like:

$$\left. \begin{aligned} \alpha_1(t) &= \alpha, \\ \beta_1(t) &= -\lambda_n, \\ \varphi_1(t) &= \alpha T_{env}, \\ \alpha_2(t) &= 0, \\ \beta_2(t) &= 1, \\ \varphi_2(t) &= 0, \end{aligned} \right\} \quad (22)$$

Recently, researches are carried out on the possibility of intensifying the cooling of pipes by withdrawing a portion of the heat flux from their inner side (Fig. 6b) [14]. In this case, to the boundary conditions (21) the following conditions should be added

$$\alpha_1(T - T_{env,1}) - \lambda_1 \frac{\partial T}{\partial r} \Big|_{r=R} = 0.$$

In solving Eq. (14) for the tubular article the coefficients at derivatives and right side of Eq. (5) take

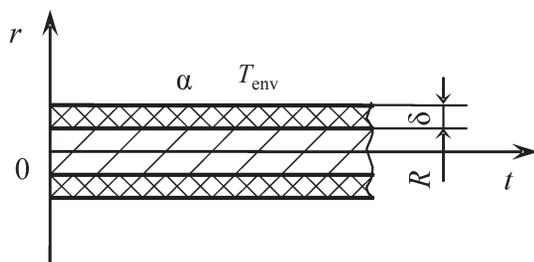


Fig. 7. Scheme of heat processing of rod article in the convective device: ( $r, R$ ) the current radius and outer radius of the core, m.

the form:

$$C(T, t, r) = \rho_1 c_1; k(T, t, r) = \lambda_1; A(T, t, r) = \lambda_1 / r; f(T, t, r) = 0.$$

In view of (17) and (19) the coefficients of the boundary conditions (7) and (8) of Eq. (5) look like:

$$\alpha_1(t) = \alpha_1, \beta_1(t) = -\lambda_1, \varphi_1(t) = \alpha_1 T_{env.1};$$

$$\alpha_2(t) = \alpha_n, \beta_2(t) = \lambda_n, \varphi_2(t) = \alpha_n T_{env.n}.$$

A mathematical model of the cooling of the rod material (such as electrical wire or cable, at least with one layer of insulation) also should be viewed in a cylindrical coordinate system (Fig. 7).

The transient heat conduction equation in this case has the form of (14), the initial condition, of (15), and to the boundary conditions (21) the following condition is added

$$\left. \frac{\partial T}{\partial r} \right|_{r=0} = 0. \quad (23)$$

Accounting for (21) and (23) the coefficients of the boundary conditions (7) and (8) of Eq. (5) take the form of (22).

Heat transfer coefficients  $\alpha_1$ ,  $\alpha_n$ ,  $\alpha$ , and  $\alpha_d$  are calculated by the relevant dependencies [15]. In this case, the following dependencies of thermal physical properties of air and water (indices “air” and “water”, respectively) on the temperature ( $^{\circ}\text{C}$ ) can be used:

– conductivity,  $\text{W m}^{-1} \text{K}^{-1}$ :

$$\lambda_{\text{air}} = 0.0244309 + 7.66687 \times 10^{-5} T - 7.32268 \times 10^{-9} T^2 - 2.83328 \times 10^{-11} T^3;$$

$$\lambda_{\text{water}} = 0.553512 + 2.38219 \times 10^{-3} T - 1.06119 \times 10^{-5} T^2;$$

– kinematic viscosity,  $\text{m}^2 \text{s}^{-1}$ :

$$\begin{aligned} \gamma_{\text{air}} &= 1.32803 \times 10^{-5} + 8.6267 \times 10^{-8} T \\ &\quad + 1.42923 \times 10^{-10} T^2; \\ \gamma_{\text{water}} &= 1.50178 \times 10^{-6} - 2.60828 \times 10^{-8} T \\ &\quad + 1.76499 \times 10^{-10} T^2 - 3.97899 \times 10^{-13} T^3; \end{aligned}$$

– Prandtl number:

$$\text{Pr}_{\text{air}} = 0.7;$$

$$\text{Pr}_{\text{water}} = 12.3048 - 0.319259 T$$

$$\begin{aligned} &+ 3.75256 \times 10^{-3} T^2 - 2.00435 \times 10^{-5} T^3 \\ &+ 3.94248 \times 10^{-8} T^4; \end{aligned}$$

– coefficient of thermal expansion,  $\text{K}^{-1}$ :

$$\beta_{\text{air}} = 1/(T + 273);$$

$$\begin{aligned} \beta_{\text{water}} &= -1.44837 \times 10^{-4} + 0.11229 \times 10^{-4} T \\ &\quad - 4.78853 \times 10^{-8} T^2 - 1.26234 \times 10^{-10} T^3. \end{aligned}$$

The temperature distribution for the current time is calculated by numerical methods (e.g., the grid method) solving the sets of equations considered above for each type of material being processed.

The method of calculation of the heat processing is as follows.

The initial data for the calculation are: the type of calculation (test or project), heat processing conditions (free or enforced convection), the type of a heat transfer agent (usually air or water), direction of article motion (horizontal, vertical, at angle to the horizontal, curved), the initial temperature of products, the velocity of article motion; main sizes of article (the thickness of rolled articles, round rod diameter, diameter and thickness of a wall of tubular articles, etc.), step of the calculation over time, temperature of heat transfer agent, heat transfer agent properties (thermal conductivity, kinematic viscosity, Prandtl number) as a function of temperature, thermal properties of material of the treated article (density, mass specific heat, thermal conductivity) as a function of temperature (or the corresponding properties of the components of the material, as well as its qualitative and quantitative composition), in the case of the enforced convection the rate of heat transfer agent flow and direction of its motion relative to the article; and for the test calculation, the length of the heat processing section, and for the project calculation, the permissible

final temperature of the material of the treated article (average or maximum over the cross-section).

Further using one of the known numerical methods of solution of linear partial differential equations of second order, e.g., the finite difference method an appropriate equation with initial and boundary conditions are solved depending on the article shape and heat processing conditions.

In the case of the test calculation a total time and current time of the heat processing are computed while in the project calculation, the current length of the heat processing section.

Then in a current step calculation of the temperature field of the article occurs.

Determination of the temperature field of the article should be continued in the next step of the calculation when the current time is less than the total time of the heat processing for the test calculation or when the current average (or local) temperature is higher than the permissible final temperature for the design calculation.

Based on the results an analysis is carried out: when the calculated final average (or local) temperature of the article for the test calculation or calculated length of the heat processing part for the design calculation is higher than the correspondent permissible value then the onset data should be varied: the temperature and/or movement mode of the heat transfer agent, type of the heat transfer agent or article velocity. Afterward the calculation is repeated at the new initial conditions.

Figure 8 shows final distribution of the temperature over the thickness of a wood filled polyethylene sheet (thickness 5 mm) calculated according to the described technique after its cooling in a roller bed of a line for production of wood polymer sheets LDPL-1200. Figure 9 shows final temperature distribution over a wire radius with copper core of 1 mm diameter and a PVC insulation of 2 mm thickness after the upgraded cooling device of a line of application of a plastic insulation on wires and cables such as LEC-63 of manufacture JSC "Bolshevik," Kiev, Ukraine.

## EXPERIMENTAL

In order to test the adequacy of the developed models to the real process we conducted complex experimental study of the process of producing of wood-plastic sheets on the equipment of the National Technical University

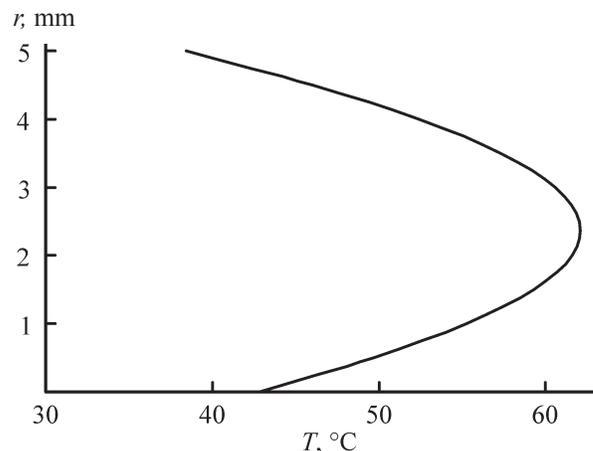


Fig. 8. Distribution of temperature over the thickness of wood-plastic sheet after its cooling in the roller conveyor.

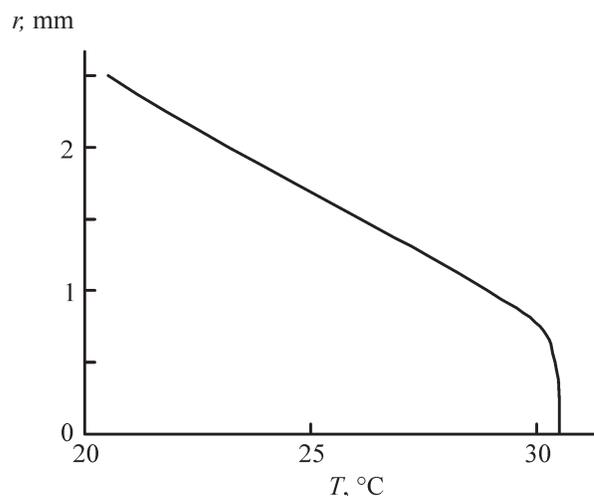
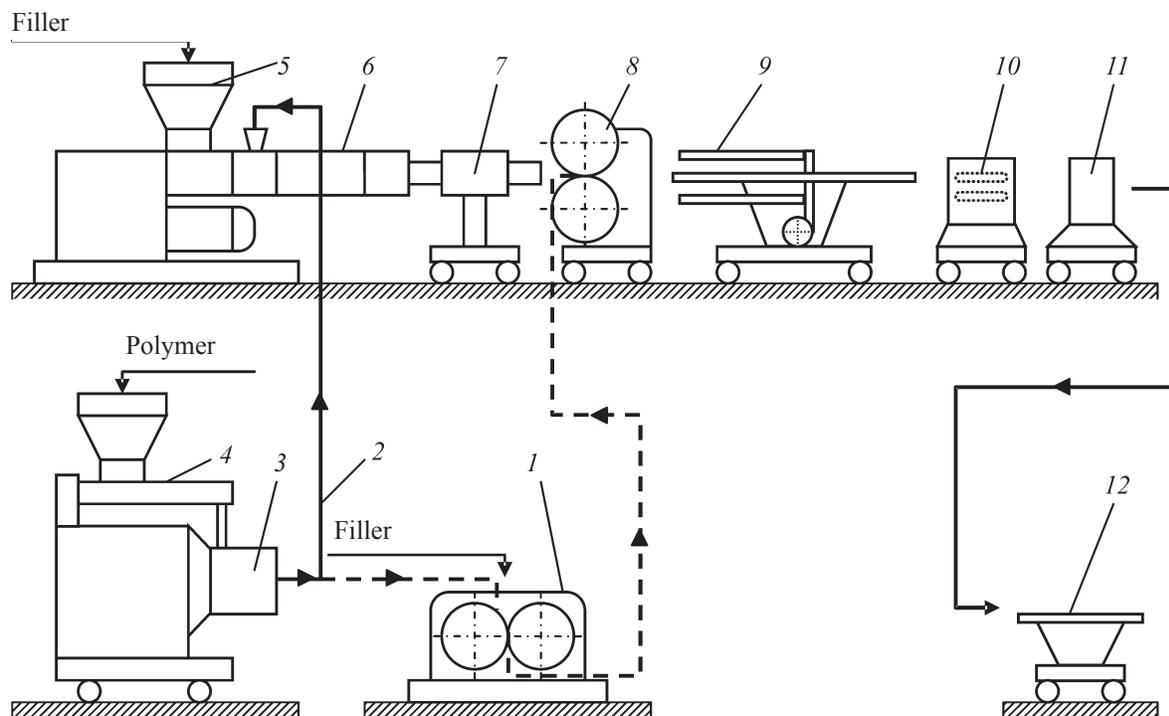


Fig. 9. Distribution of temperature over the radius of a wire with copper core and PVC insulation and after its cooling in a water bath.

of Ukraine "Kyiv Polytechnic Institute" (Fig. 10).

We studied samples of sheets produced by a roll setup after their calibration, whose ratio of width and length to their thickness was higher than 10 and due to this fact heat transfer over the width and length of the samples can be ignored and the cooling process can be considered as one-dimensional [12].

Experimental values of temperature of the sheet material (material composition: 60 wt% of secondary LDPE and 40 % wt of oak sawdust) were determined by thermocouples placed at a depth of 0.5 mm from the surface of the sheet, as well as along their center. The studies were conducted for sheet of thickness 3, 6, and



**Fig. 10.** Scheme of the experimental production line for manufacturing wood-plastic sheets: (1) mix-warming rolls, (2) melt conduit, (3) disk extruder, (4) polymer feed, (5) filler batcher, (6) a single screw extruder, (7) extrusion head, (8) glazing calender, (9) cooling device, (10) a pulling device, (11) cross cutting device, (12) storage.

10.1 mm.

An error of temperature measurement in conducting experiments was

$$T = \bar{T} \pm \sigma(\bar{x}) = \bar{T} \pm 0.4^{\circ}\text{C},$$

and that of thickness of wood-plastic sheets was

$$h = \bar{h} \pm \sigma(\bar{h}) = \bar{h} \pm 0.004 \text{ mm}.$$

A deviation of theoretical values of temperature from experimental was no more than 10–11%.

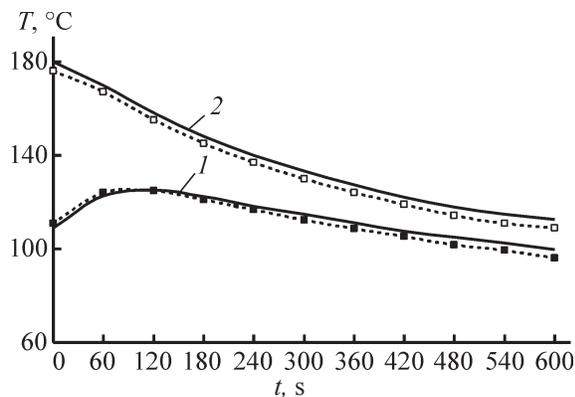
After the release of the samples from the roll space of a glazing calender and their further cooling in the air the temperature of the surface layers of the sheet first increases due to heat transfer from its inner layers and then the cooling of the surface and inner layers occurred simultaneously (Fig. 11).

To intensify the cooling of the sheet materials of thickness 6 mm the process can be carried out under conditions of forced convection, for example, by air blowing into device of tunnel type (Fig. 12).

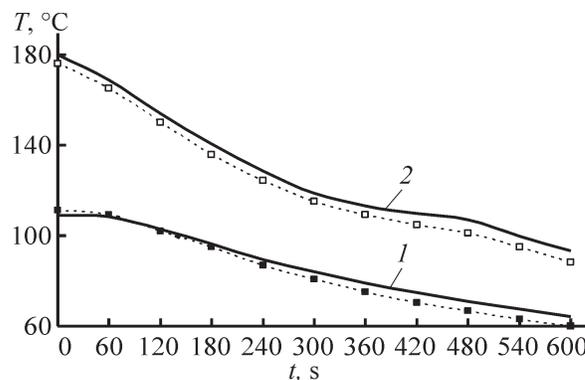
In turn, the intensification of the cooling can lead to different conditions of structure formation of the sheet material in its different layers and, as a consequence, uneven properties of the finished product, in particular, to arising excessive residual stresses that adversely affect the operational performance of the product. Thus, the intensification of the cooling process of thick-walled sheets reduces the cooling time, but can adversely affect the quality indicators of the finished product and can not be practical.

## CONCLUSIONS

Commercial operation of production lines for the manufacture of various articles from thermoplastic materials (homogeneous wood-plastic sheets and wood-plastic sheets with unilateral decorative coating, horizontal long wood-plastic materials, single and multilayer plastic pipes, and electric wires and cables with single- and multi-layer coating) confirm the adequacy of the suggested models to the actual process of heat treatment. Taking into account the perspective of using continuously molded materials and articles made



**Fig. 11.** Variation of temperature of a wood-plastic sheet of thickness 10.1 mm over its thickness vs. time in the air under free convection: (1) sample surface (calculation); (2) sample centre (calculation); (■) sample surface (experiment); (□) sample centre (experiment).



**Fig. 12.** Variation of temperature of a wood-plastic sheet of thickness 10.1 mm over its thickness vs. time in the air under enforced convection: (1) sample surface (calculation); (2) sample centre (calculation); (■) sample surface (experiment); (□) sample centre (experiment).

of composite materials, it is useful to study the heat processing of profiles with different cross-section.

#### DESIGNATIONS

$c_p$  is mass isobaric heat capacity,  $J\ kg^{-1}\ K^{-1}$ ;  
 $q$  is surface heat flow,  $W\ m^{-2}$ ;  
 $q_V$  is volume density of the heat flow of internal energy sources,  $W\ m^{-3}$ ;  
 $Pr$  is Prandtl number;  
 $r$  is current radius,  $m$ ;  
 $R$  is inner radius of a drum or a pipe article,  $m$ ;  
 $R_d$  is outer drum radius,  $m$ ;  
 $t$ , time,  $s$ ;  
 $T$ , temperature,  $^{\circ}C$ ;  
 $W$ , linear velocity,  $m\ s^{-1}$ ;  
 $x, y, z$ , are Cartesian coordinates;  
 $\alpha$  is heat transfer coefficient,  $W\ m^{-2}\ K^{-1}$ ;  
 $\alpha_1$  and  $\alpha_2$  are coefficients of equations of boundary conditions;  
 $\beta$  is coefficient of thermal expansion,  $1\ K^{-1}$ ;  
 $\beta_1$  and  $\beta_2$  are coefficients of equations of boundary conditions;  
 $\delta$  is the total thickness of the material (article) or thickness of its individual layer,  $m$ ;  
 $\lambda$  is thermal conductivity,  $W\ m^{-1}\ K^{-1}$ ;  
 $\nu$  is kinematical viscosity,  $m^2\ s^{-1}$ ;  
 $\rho$  is density,  $kg\ m^{-3}$ ;  
 $\varphi_1$  and  $\varphi_2$  are coefficients of equations of boundary conditions;  
 $\nabla$  is Hamilton operator.

**Main indices:** 0 for the onset state;  $i$  for ordinal

index of a layer of the multilayer article ( $i = \overline{1, n}$ );  $d$  for the outer surface of the drum; water for water; air for air;  $f$  for final;  $in$  for initial;  $t$  for heat transfer agent;  $env$  for environmental;  $tpm$  for thermoplastic material.

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