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PROCESSES AND DEVICES
OF CHEMICAL MANUFACTURES

Simulation of Processing the "Power" Composition in a Mixer with Oval Rotors

I. O. Mikulionok

Ukrainian Technical University "Kyiv Polytechnical Institute", Kiev, Ukraine
e-mail: i.mikulionok@kpi.ua

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Abstract— The main dependence for determining the energy-power parameters of processing the compositions, whose behavior under load is described by a power rheological law in the operation gap of mixers with oval rotors. In the proposed method of calculating the power of the mixer drive motor an attempt was made to account for movement of a treated material not only in circular but also in the axial direction of the mixing chamber. These dependences for determining the pressure and power supplied to the rotors for rotation thereof may be recommended for engineering calculations of mixing equipment for plastics and rubber mixtures.

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Rotary mixers are one of the main types of equipment in manufactures of products based on macromolecular compounds: tires, rubber products, conveyor belts, linoleum, etc. A body of the rotory mixer comprises a chamber in a cross-section made in the form of eight. In every part of the chamber two rotors, more often in the form of transformed oval, rotate toward each other. That is why these mixers are also known as mixers with oval rotors [1–8] (Fig. 1).

An operation part of the every oval rotor of the mixer was made in a form of an intermittent helix thereby in any cross-section the specified part represents an oval tapering in one direction. A surface of each of the rotors is formed by two helical blades, one of which, the longer, is of an angle of ascent helix $\alpha_l = 60º$, and another, the short blade, of $\alpha_s = 60º$. An angle of blade twisting is usually 90º. The length of the short blade l_1 (more precisely the length of its projection on the center plane of the rotor) is from about 38 to 43% of the total length of the working part of the rotor l (the length of the mixing chamber), and the length of the long blade l_2, approximately 70% of the value l. The helix of the short blade has a left direction, and one of the long blade, the right direction (Fig. 2).

Considered rotor configuration promotes the mixing effect. The location of the rotors in the chamber is following: the long blade of one rotor is located against a short blade of second one. Because of this, as well as different speed of the rotors, the processed mixture in the chamber moves by a complicated trajectory of figure of eight.

The rotors rotate towards one another in the space bounded by a wall of the working chamber (half-chamber of both rotors), as well as by upper and lower

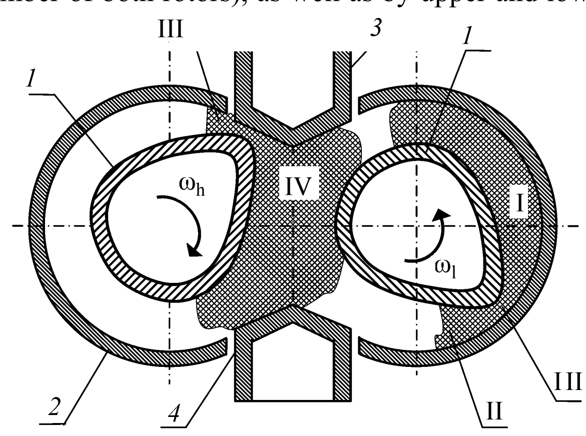

Fig. 1. Scheme of processing the composition in the mixer chamber. (*1*) Rotors; (*2*) operation chamber; (*3*, *4*) upper and lower closures; (I–VI) characteristic regions of strain mixtures; (ω_h) and (ω_l) angular velocity of a high-speed and low speed rotors, rad/s.

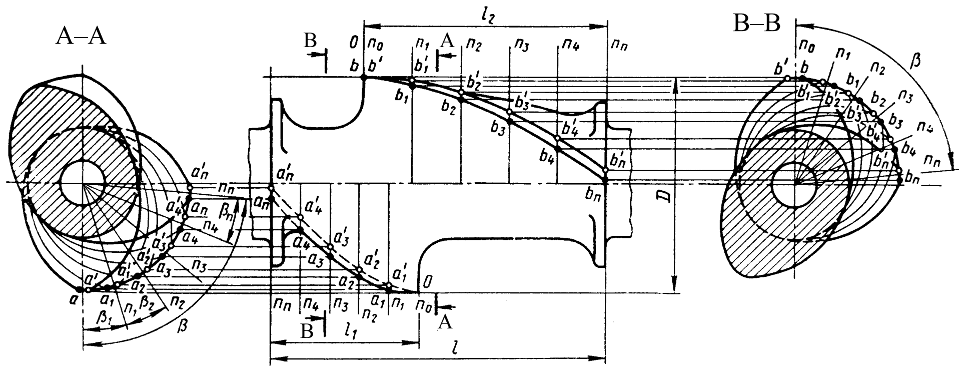

Fig. 2. Construction of a profile pf the working part pf the mixer rotor [1]: (n_i, β_i, a_i, b_i) beams, central angles, points and their projections, respectively.

gates. A working volume of the mixing chamber can be conventionally divided into four characteristic zones: the sickle-shape area I of the effective mixing created by the wall of one of the half-chambers of the mixing chamber and the anterior (front) surface of the corresponding rotor, the area II, between the rotors, in which the redistribution of the treated mixture occurs between the half-chambers, and the occipital region III, in which the material enters after passing through the area IV, the area of a minimum gap between the wall of the half-chamber and a crest of the rotor [9] (Fig. 1).

Under the action of the rotating rotor the main process of mixing the composition and dispersion of its components occurs in region II. Therewith the composition moves not only in the circular direction, and along the corresponding rotor: along the helical surfaces of its blades. Thus, if in a circular direction the mixture moves under the action of pressure developed in the bulk of composition as a result of its deformation [1–6] we henceforth assume that in the axial direction there is a free-flow movement of the composition. Also we consider the free-flow movement of the composition as its motion in space bounded by a finite set of channels formed by parallel walls (the height of these channels change from the maximum height that equals the difference between the radii of the inner surface of the half-chamber and base of the rotor to the minimum height that corresponds to the value of the minimum gap between the inner surface of the half-chamber and the crest of the rotor): thus we apply the stepped approximation [10].

Rotary mixers are one of the most material and power consuming equipment (their mass reaches almost 200 t, and the power of the drive motor, 4 MW [8]), so determination of power parameters of the process, primarily of power consumed by mixer drive motor is of particular importance.

Mainly in simulation of composition processing in the mixer with oval rotors it is assumed that [2–6, 11] treated material is incompressible; the material flow is steady and one-dimensional; a depth of the working gaps in the central cross-section of the rotors is small compared with the radius of the rotors and the half-chambers; mass and inertial forces are negligible compared with viscous forces; analysis of the material flow is conducted by deploiment on the plane of the working surface of the rotor and half-chamber in the region of the sickle-shape gap.

These techniques were focused on the calculation of the process for preparing a composition flowing in accordance with a power rheological law [3, 4, 6], or the Newton's law of viscosity [4, 11] (taking into account the effective viscosity of the material). At the same time in one of the most acceptable methods of a calculation developed by R.V. Turner and M.S. Akutin [5] the process was taken into account only in the sickle-shape gaps, and the impact of minimum gaps, where the highest shear stress occurs, was neglected.

In addition, calculated by these techniques the values of power consuming for mixing, which take into account the movement of material in the mixing chamber only in the circular direction of the rotors [3–6], usually were by 10–15% less than the values of power defined directly under the operational conditions in the industrial equipment [1, 9].

Thus, there is currently no comprehensive method of calculating the rotary mixer, which would provide a definition of the power parameters of process (the drive power of rotors, the parameters of the cooling system of working bodies) accounting for the mutual influence of the sickle-shape and minimum gaps, a dependence of the speed of the front surface of the rotor on radius, as well as the variation of temperature of the material, which is of great importance in the case of processing the heat-sensitive compositions.

To simplify the modeling process, we take into account the above assumptions, but the analysis of the material flow we conduct by deploimeny on the plane of the working surface of the rotor in the regions of consequently placed maximum gap (created by the base of the rotor and the wall of appropriate half-chamber), sickle-shape, and minimum gaps (Fig. 3).

Since, during the steady process of mixing a load factor of the working chamber is 0.47...0.85 [1, 8], the intense deformation of the treated composition at the maximum gap (Fig. 1) does not occur, and the active effect of the rotors on the composition takes place only in the sickle-shape and minimum gaps.

Accounting for the above assumptions the equations simulating the flow of the material described by the power rheological law in the sickle-shape and minimum gaps between the surfaces of the rotor and the corresponding half-chamber (equation of motion, continuity, and the rheological equation), take the form:

$$-\frac{dp}{dx}+\frac{\partial \tau_{xy}}{\partial y}=0; \quad (1)$$

$$G_V=\int_0^h w_x dy; \quad (2)$$

$$\tau_{yx}=K|\dot{\gamma}|^n \operatorname{sign}(\dot{\gamma}) \text{ or}$$

$$\tau_{yx}=K_0 \exp\left(-\beta\frac{T-T_0}{T_0}\right)\left|\frac{\partial w_x}{\partial y}\right|^n \operatorname{sign}\left(\frac{\partial w_x}{\partial y}\right), \quad (3)$$

Boundary conditions over the speed in the sickle-shape and minimum gaps:

$$w_x\big|_{y=0}=W_{\text{crest}}(x); \qquad w_x\big|_{y=h}=0; \quad (4)$$

$$w_x\big|_{y=0}=W_{\text{crest}}; \qquad w_x\big|_{y=h}=0, \quad (5)$$

where $W_f(x) = \omega_{b(t)} R_p(x)$; $R_p(x)$ is radius of the frontal surface of the rotor as function of coordinate x.

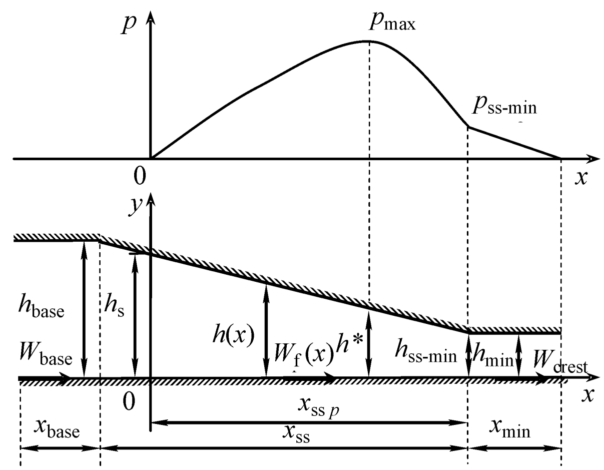

Fig. 3. Scheme of the maximum, sickle-shape, and minimum gaps after deploiment. (x, y) Current coordinates along and across channel, m; (p, p_{max}) current and maximum pressure in the channel, Pa; (p_{ss-min}) pressure in the point of passing the sickle-shape gap in the minimum gap, Pa; (x_{base}, x_{ss}, x_{min}) length of the maximum (on the section of the rotor base), sickle-shape and minimum (in the section of the rotor crest) gaps, m; ($x_{ss.p}$) length of active section of the sickle-shape gap (the section where pressure p occurs), m; (h_{base}, h, h_{min}) maximum (starting), current, and minimum heights of the channel, m; (h_{ss-min}) the gap height in the point of passing the sickle-shaped gap in the minimum gap, m; (h_s, h^*) the gap height in the beginning of the section of an active deformation of the material in the sickle-shape gap and in the point of passing the sickle-shape gap in minimum gap, m; (W_{base}, W_f, W_{crest}) linear speed of the working surface of the rotor (the base, the frontal part, and crest), m/s.

After substitution of relations (3) in the equation of motion (1) and its further integration accounting for the boundary conditions (4) and (5) for the sickle-shape and minimum gaps we can obtaine:

$$w_x=W_{\text{crest}}(x)\left(1-\frac{y}{h}\right)-\left(\frac{1}{K}\frac{dp}{dx}\right)^{1/n}\frac{n}{1+n}(yh^{1/n}-y^{1/n+1}); \quad (6)$$

$$w_x=W_{\text{crest}}\left(1-\frac{y}{h}\right)-\left(\frac{1}{K}\frac{dp}{dx}\right)^{1/n}\frac{n}{1+n}(yh^{1/n}-y^{1/n+1}). \quad (7)$$

Let us consider first the sickle-shaped gap.

Taking into account (2) and (6), we get

$$G_V=\int_0^h\left[W_{\text{crest}}(x)\left(1-\frac{y}{h}\right)-\left|\frac{1}{K}\frac{dp}{dx}\right|^{1/n}\frac{n}{1+n}(yh^{1/n}-y^{1/n+1})\right]dy,$$

and after integration

$$G_V=\frac{1}{2}W_{\text{crest}}(x)h-\left|\frac{1}{K}\frac{dp}{dx}\right|^{1/n}\frac{nh^{1/n+2}}{2(1+n)(1+2n)}. \quad (8)$$

We introduce the dimensionless analogs of x and y transforming the sickle-shaped gap in a rectangular [9]:

$$\xi = \frac{xm}{h_s}, \ \varepsilon = \frac{y}{h}, \tag{9}$$

where $m = (h_s - h_{min})/x_{ss\,p}$ (Fig. 3).

In new coordinate system the gap height at a distance from the beginning of the sickle-shaped gap is determined by relationship $h = h_s(1 - \xi)$.

Accounting for Eqs. (9) equation (8) takes the form

$$G_V = \frac{1}{2} W_{crest}(\xi) h_s (1-\xi) - \frac{h_s^{1/n+2} n}{2K^{1/n}(1+n)(1+2n)} (1-\xi)^{1/n+2} \left| \frac{m}{h_s} \frac{dp}{d\xi} \right|^{1/n}.$$

Then pressure gradient is:

$$\frac{dp}{d\xi} = \frac{K(1+n)^n (1+2n)^n}{mn^n h_s^{2n} (1-\xi)^{2n+1}} \left| h_s(1-\xi) W_{crest}(\xi) - 2G_V \right|^n \times \operatorname{sign}\,[h_s(1-\xi) W_{crest}(\xi) - 2G_V]. \tag{10}$$

Substituting Eq. (10) in Eq. (6) and using the change of variables in (9), we obtain the dependence for determining the velocity field in the sickle-shaped gap

$$w_x = W_{crest}(\xi) \times \left(1 - \varepsilon - \varepsilon(1-\varepsilon^{1/n}) \frac{[h_s(1-\xi) - 2G_V / W_{crest}(\xi)](1+2n)}{h_s(1-\xi)}\right).$$

Then the speed gradient is

$$\frac{\partial w_x}{\partial y} = \frac{W_{crest}(\xi)}{h_s(1-\xi)} \times \left[-1 - \frac{(1+2n)}{h_s(1-\xi)} \left(h_s(1-\xi) - \frac{2G_V}{W_{crest}(\xi)} \right) \left(1 - \left(\frac{1+n}{n} \right) \varepsilon^{1/n} \right) \right]. \tag{11}$$

Integrating Eq. (10) in the interval [0; ξ] we can determine the pressure distribution in the sickle-shaped gap

$$p = \frac{K(1+n)^n (1+2n)^n}{mn^n h_s^{2n}} \int_0^{\xi} \left| h_s(1-\xi) W_{crest}(\xi) - 2G_V \right|^n \times \operatorname{sign}[h_s(1-\xi) W_{crest}(\xi) - 2G_V] \frac{1}{(1-\xi)^{2n+1}} d\xi. \tag{12}$$

Considering Eq. 10 we can show that at the current value ξ and the depth of the gap $h \geq 2G_V/W_{crest}(\xi)$ the pressure in the gap increases, and at $h < 2G_V/W_{crest}(\xi)$, decreases. The value h^* for analysis of the process in the rotor mixer (mainly for the case of $n \leq 0.25$) approximately can be determined by the following relationship [11]:

$$\frac{\sqrt{(1+n)/n}}{2^{1+n}} \left(\frac{h_s}{h_f} - \frac{1+2n}{2n} \right) + \frac{2^n}{1+2n} \left[1 - \left(\frac{2h_f}{h^*} - 1 \right)^{-n} \right] + \frac{2^n n}{1+2n} \left[\left(\frac{1+n}{n} \right)^n - 1 \right] - \frac{2^{1+n} n^{2+n}}{(1+n)(1+2n)^{1+n}} \left[\left(\frac{1+n}{n} \right)^{1+n} - 1 \right] = 0.$$

or by expression $h^* = 2h_s h_{min}/(h_s + h_{min})$ [12] or $h^* = 1{,}67 h_{min}$ [11].

Now we consider the minimum gap.

Through a unit width of the minimum gap formed by the crest of the rotor and the wall of the half-chamber, due to the continuity of the flow the material passes with a flow rate corresponding to the flow rate in the sickle-shaped gap. Taking into account Eqs. (2) and (7), we obtain

$$G_V = \frac{1}{2} W_{crest} h_{min} - \left| \frac{1}{K} \frac{dp}{dx} \right|^{1/n} \frac{n h_{min}^{1/n+2}}{2(1+n)(1+2n)}.$$

Then the pressure gradient along the minimum gap is

$$\frac{dp}{dx} = \frac{K}{h_{min}^{1+2n}} \left(\frac{(1+n)(1+2n)}{n} \right)^n \times \left| W_{crest} h_{min} - 2G_V \right|^n \operatorname{sign}(W_{crest} h_{min} - 2G_V). \tag{13}$$

Since the pressure at the inlet of the material in the minimum gap is p_{ss-min}, and at its output the excess pressure equals zero, the expression for the pressure at the boundary of the sickle-shaped and minimum gaps will have the form:

$$p_{ss-min} = \frac{K}{h_{min}^{1+2n}} \left(\frac{(1+n)(1+2n)}{n} \right)^n \left| W_{crest} h_{min} - 2G_V \right|^n x_{min}.$$

It should be noted that the condition for joining the two consecutive gaps: sickle-shaped and the minimum, is the equality of pressure at the outlet of the first and at the inlet to the second ($p = p_{ss-min}$), hence the length of the active deformation zone of the material in sickle-shaped gap x_{ss} is determined by the method of the successive approximations (an onset of the zone of active deformation, i.e., the beginning of the coordinate system, depending on the mode of processing and technological properties of the material is “floating”). In the first approximation we can assume $h_s = h_{base}$ and $x_{ss\,p} = x_{ss}$ with subsequent refinement of the values h_s, and $x_{ss\,p}$.

Let us define the spacing forces exerted by the material being deformed on the rotor and the wall of the respective half-chamber.

To determine the spacing forces acting in the sickle-shaped gap per unit length of the rotor blade, we integrate Eq. (10) over its working surface:

$$F_{ss} = \int_0^{x_{ss\text{-}p}} p(x)\,dx = \frac{h_s}{m}\int_0^{\xi_{ss\text{-}p}} p(\xi)\,d\xi.$$

Substituting this expression in Eq. (12) we get

$$F_{ss} = \frac{K(1+n)^n(1+2n)^n}{m^2 n^n h_s^{2n\text{-}1}}$$

$$\times \int_0^{\xi_{ss\,p}}\int_0^{\xi} \left| h_s(1-\xi)W_{crest}(\xi) - 2G_V \right|^n \frac{1}{(1-\xi)^{2n+1}}\,d\xi\,d\xi.$$

By analogy, we define the spacing forces acting in the minimum gap using Eq. (13):

$$F_{min} = \frac{K}{2h_{min}^{1+2n}}\left(\frac{(1+n)(1+2n)}{n}\right)^n \left| W_{crest}h_{min} - 2G_V \right|^n x_{min}^2.$$

The frictional force exerted by the material in the sickle-shaped gap per unit length of the rotor blade over a tangent to its surface taking into account Eqs. (3) and (11) is

$$P_{ss} = \int_0^{x_{ss\,p}} \tau_{ss\,|y=0}\,dx = \frac{h_s}{m}\int_0^{\xi_{ss\,p}} \tau_{ss\,|\varepsilon=0}\,d\xi$$

$$= -\frac{K}{mh_s^{n-1}}\int_0^{\xi_{ss\,p}} \left| -W_f(\xi) - \frac{(1+2n)}{h_s(1-\xi)}\left(h_s(1-\xi)W_f(\xi) - 2G_V\right) \right|^n$$

$$\times \operatorname{sign}\left(-W_f(\xi) - \frac{(1+2n)}{h_s(1-\xi)}\left(h_s(1-\rho)W_f(\xi) - 2G_V\right) \right)\frac{1}{(1-\xi)^n}\,d\xi.$$

The specific frictional force exerted by the composition in the minimum gap on the rotor over the tangent to the crest surface can be computed fairly accurately by the following expression

$$P_{min} = \int_0^{x_{rp}} \tau_{min\,|y=0}\,dx = Kx_{min}\frac{W_{crest}^n}{h_{min}^n}.$$

The total spacing forces acting on the each of the rotor can be determined by technique reported in [13].

The specific dissipation power whose value is required in computation of the cooling systems of the rotors and chamber is determined by Eq. (14)

$$q_{dis} = \tau_{yx}\left|\frac{\partial w_x}{\partial y}\right|. \quad (14)$$

In view of Eq. (11) for the sickle-shaped gap

$$q_{dis\,ss} = \frac{KW_{crest}(\xi)^{n+1}}{h_s^{n+1}(1-\xi)^{n+1}}$$

$$\times \left| -1-(1+2n)\left(1-\frac{2G_V}{h_s W_{crest}(\xi)(1-\xi)}\right)\left(1-\frac{1+n}{n}\varepsilon^{1/n}\right) \right|^{n+1}. \quad (15)$$

Integrating this equation over the entire volume of material deformed into the sickle-shaped gap, we can find the total dissipation power in this gap

$$Q_{dis\,ss} = \int_0^{L_{l(s)}}\int_0^{h}\int_0^{x_{ss\,p}} q_{dis\,ss}\,dx\,dy\,dz,$$

where z is a coordinate directed along the rotor axis, m; $L_{l\,(s)}$, the total length of the long or short blade of the rotor along the helical line, m:

$$L_{l\,(s)} = N_{l\,(s)}\frac{2\pi R_{crest}}{\cos\alpha_{l\,(s)}},$$

where $N_{l\,(s)}$ is an amount of the complete revolutions of the corresponding blade (for the twist angle of the blades 90º $N = 0.4$).

Takung into account Eqs. (14) and (15) we obtain

$$Q_{\text{dis ss}} = \frac{(L_l + L_s)K}{m h_s^{n-1}}$$

$$\times \int_0^1 \int_0^{\xi_{ss\,p}} \left| -W_{\text{crest}}(\xi) - \frac{(1+2n)}{h_s(1-\xi)}(h_s(1-\xi)W_{\text{crest}}(\xi) - 2G_V)\left(1 - \left(\frac{1+n}{n}\right)\varepsilon^{1/n}\right)\right|^{n+1} \frac{1}{(1-\xi)^n}\, d\xi\, d\varepsilon \quad (16)$$

The distribution of the dissipation power in the sickle-shaped gap between the rotor and chamber (for further calculation of their cooling system) can be performed in the case of integration of Eq. 16 over coordinate ξ in the ranges of (0; 0.5) and (0.5; 1), respectively.

In the minimum gap the power spent on the material deformation with a sufficient accuracy can be determined by the following dependence

$$Q_{\text{dis min}} = (L_l + L_s)Kx_{\min} \frac{W_{\text{crest}}^{n+1}}{h_{\min}^n}. \quad (17)$$

Now we define the component of the power spent on the material deformation when it moving under the influence of the helical blades in the axial direction of the rotors.

For calculation of the power expended on the movement of the treated material in the axial direction of the rotors we use an approach supposed for the analysis of the free liquid flow that is described by the power rheological equation in the channel bounded by the parallel walls, one of which is mobile [11].

By analogy with Eq. (17) the power expended in the course of the material movement along the axis of the rotor can be determined from the dependence (separately for long and short blades of the appropriate rotor: a fast- or slow-speed)

$$Q_{\text{dis ax}} = \int_0^{x_{ss\,p}} L_{l(s)} K \frac{W(x)_{\text{r ax f (sl)}}^{n+1}}{h(x)^n}\, dx, \quad (18)$$

where $W(x)_{\text{r ax f (sl)}}$ is the speed in the axial direction of the material on the surface of the fast or slow-speed rotor in the considered point; dx, the section width for stepped approximation.

The value of the speed $W(x)_{\text{r ax}}$ in each a point can be computed with the aid of the approach used for an nalysis of the worm extrusion [10].

Accounting for, that $W(x)_{\text{r f (sl)}} = \omega_{\text{f (sl)}} R_r(x) = \omega_{\text{f (sl)}} (R_{\text{crest}} - h(x))$, where $h(x)$ is height of the sickle-shaped gap in the considered point, value $W(x)_{\text{r ax}}$ is calculated by the following dependence

$$W(x)_{\text{r ax f (sl)}} = W(x)_{\text{r ax (sl)}} \sin\alpha_{1(s)} \cos\alpha_{1(s)}$$

$$= 0.5 W(x)_{\text{r f (sl)}} \sin(2\alpha_{1(s)}) = 0.5\omega_{\text{f (sl)}}(R_{\text{crest}} - h(x))\sin(2\alpha_{1(s)}). \quad (19)$$

Accounting for Eq. (19) equation (18) takes the form

$$Q_{\text{dis ax}} = L_{1(s)} K\, [0.5\,\omega_{\text{f (sl)}} \sin(2\alpha_{1(s)})]^{n+1}$$

$$\times \int_0^{x_{ss\,p}} \frac{\left[R_{\text{crest}} - \left(h_s - \frac{h_s - h_{\min}}{x_{ss}} x\right)\right]^{n+1}}{\left[h_s - \frac{h_s - h_{\min}}{x_{ss}} x\right]^n} dx.$$

The common values of the dissipation power in the half-chambers with placed inside the fast and slow-speed rotors are

$$Q_{\text{dis f}} = (Q_{\text{dis ss}} + Q_{\text{dis min}} + Q_{\text{dis ax}})_{\text{f}};$$

$$Q_{\text{dis sl}} = (Q_{\text{dis ss}} + Q_{\text{dis min}} + Q_{\text{dis ax}})_{\text{sl}}.$$

Finally, the rotor drive power of the mixer can be determined by the formula

$$N_{\text{dr}} = \frac{Q_{\text{dis f}} + Q_{\text{dis sl}}}{\eta_{\text{dr}}},$$

where η_{dr} is the efficiency of the rotor drive.

The calculation of a composition in mixers 250/40 GOST 11996-79 (RSVD-140-40 [1]) and 250/20 GOST 11996-79 (RSVD-140-20 [1]) and comparison of the simulation results with the results calculated by other known methods [3, 11, 13, 14] as well as the results of measuring the power of the drive motor directly under the industrial conditions [1] were made in order to test an adequacy of the developed mathematical model of the rotary mixer.

The main mixer parameters:

– mixer 250/40 GOST 11996-79: the free volume of the mixing chamber 250 dm^3, the working volume 140 dm^3; the standard frequency of the revolution of the rotors 40 rpm and 33,5 rpm; the power of the electric motor of the rotors 800 kW;

– mixer 250/20 GOST 11996-79: the free volume of the mixing chamber 250 dm^3, the working volume 140 dm^3; the standard frequency of the revolution of the rotors 20 rpm and 17 rpm; the power of the electric motor of the rotors 315 kW;

An analysis was made for the rubber mixture of the tire manufacture with the following properties: K_0 = 80000 Pa s^n; n = 0.2 [14].

For the mixer 250/40 an error of the calculation of the drive power according to the supposed method was 3,2 %, since according to V. Krasovskii and R. Torner method, 11.5%, and by Z. Tadmor and K. Gogos [2], 30.7%. For the mixer 250/20 the noted values were 4.1, 18.8, and 37%, respectively.

Thus, a comparison of theoretical and experimental data allows a conclusion about the adequacy of the developed technique to the results of operation of industrial equipment, and the approach developed to computation of the parameters of the process in a rotary mixer with the oval rotors, in particular of power of the rotor drive can be recommended for application to designing, improvement, and operation of the equipment.

One of the criteria of quality of the thermoplastic compositions is the total shear strain γ_Σ, the optimal value of which providing the production mix of high quality and set-valued by numerous experiments, is 2000...2500 [7, 14].

Then a mixing time required for deformation accumulation γ_Σ for the material with the total weight M_Σ, into the mixer is [9]

$$t = \frac{M_\Sigma}{M}\frac{\gamma_\Sigma}{\bar{\dot\gamma}},$$

where $M_\Sigma = \varphi_{mc}V_{mc}\rho_m$; φ_{mc} is a load factor of the mixing chamber; V_{mc}, the free volume of the mixing chamber, m^3; ρ_m, the material density; M, the material weight, that is deformed in a certain time in the sickle-shaped and minimum gaps of both half-chambers (M_f and M_{sl}, respectively), kg:

$$M = M_f + M_{sl} = [(V_{ss} + V_{min})\rho_m]_f + [(V_{ss} + V_{min})\rho_m]_{sl} = 2L\rho_m(S_{ss} + S_{min}),$$

where V_{ss} and V_{min} are the volume of the sickle-shaped and minimum gaps, m^3; S_{ss} and S_{min} are the areas of the cross-sections of the sickle-shaped and minimum gaps, m^2.

For any cross-section of the sickle-shaped gap the shear rate is computed by Eq. (11)

$$\dot\gamma_{ss} = \left(\frac{\partial v_x}{\partial y}\right)_{ss} = \frac{W_f(\xi)}{h_s(1-\xi)}\left[-1-\frac{(1+2n)}{h_s(1-\xi)}\left(h_s(1-\xi) - \frac{2G_V}{W_f(\xi)}\right)\left(1-\left(\frac{1+n}{n}\right)\varepsilon^{1/n}\right)\right].$$

Then an absolute value of the average shear rate $\bar{\dot\gamma}_{ss}$ in the sickle-shaped gap is

$$\bar{\dot\gamma}_{ss} = \frac{2}{(h_s + h_{min})x_{ss\,p}}\int_0^{x_{ss\,p}}\int_0^{h}\left|\bar{\dot\gamma}_{ss}\right| dy\,dx$$

or in the dimensionless coordinates

$$\bar{\dot\gamma}_{ss} = \frac{2m^2h_s}{(h_s^2 - h_{min}^2)}\int_0^{x_{ss\,p}}\int_0^{h} W_f(\xi)\left[-1-\frac{(1+2n)}{h_s(1-\xi)}\left(h_s(1-\xi) - \frac{2G_V}{W_f(\xi)}\right)\left(1-\left(\frac{1+n}{n}\right)\varepsilon^{1/n}\right)\right] d\varepsilon\, d\xi.$$

An absolute value of the average shear rate $\bar{\dot\gamma}_{min}$ in the minimum gap can be calculated by the following expression

$$\bar{\dot\gamma}_{min} = \frac{W_{crest}}{h_{min}}.$$

The average shear rates in the half-chambers of the fast and slow-speed rotors can be calculated by relationships:

$$\bar{\dot\gamma}_f = \left(\frac{\bar{\dot\gamma}_{ss}S_{ss} + \bar{\dot\gamma}_{min}S_{min}}{S_{ss} + S_{min}}\right)_f;\ \bar{\dot\gamma}_{sl} = \left(\frac{\bar{\dot\gamma}_{ss}S_{ss} + \bar{\dot\gamma}_{min}S_{min}}{S_{ss} + S_{min}}\right)_{sl},$$

and the average sheat rate $\bar{\dot\gamma}$ in the mixing chamber, as arithmetic mean of the values $\bar{\dot\gamma}_f$ and $\bar{\dot\gamma}_{sl}$

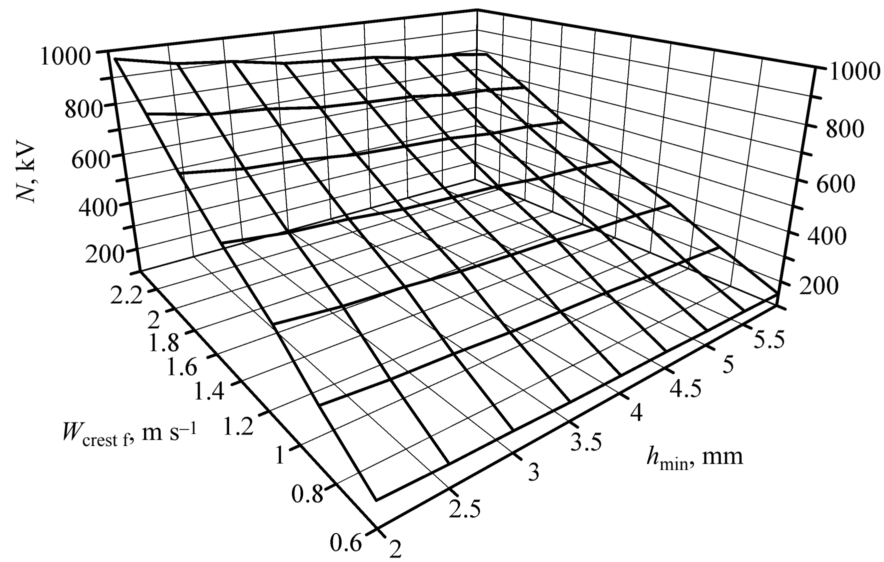

Fig. 4. The dependence of the drive power of the mixer on the depth of the minimum gap and speed of rotation of the fast rotor.

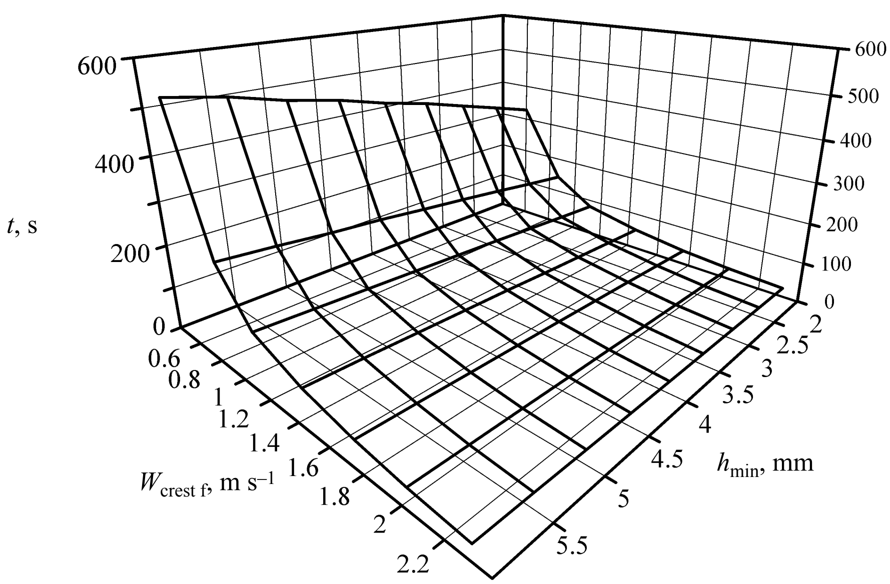

Fig. 5. The dependence of the mixing time on the depth of the minimum gap and speed of rotation of the fast rotor.

$$\bar{\dot{\gamma}} = \frac{\bar{\dot{\gamma}}_f + \bar{\dot{\gamma}}_{sl}}{2}.$$

The duration of the entire cycle of mixing that determines the mixer efficiency, is computed in view of duration of nonproductive operations.

On Figs. 4–6 the results of the numerical study of an effect of variation of the speed of rotation of the rotors and value of the minimum gap on the power parameters of the process in the rotory mixer with the volume of the mixing chamber 250 dm^3 of “NPP Bol’shevik” JSC (Kiev).

As shown from Fig. 4 an increase on the rotation speed of the fast rotor $W_{\text{crease f}}$ results in a growth of the drive power. An effect of the value of the minimum gap $h_{\min}$ is of the opposite character. However, if in the case of $W_{\text{crest f}}$ of approximately to 1 m s^{-1} the value of h_{crest} affect not essentially the drive power (about

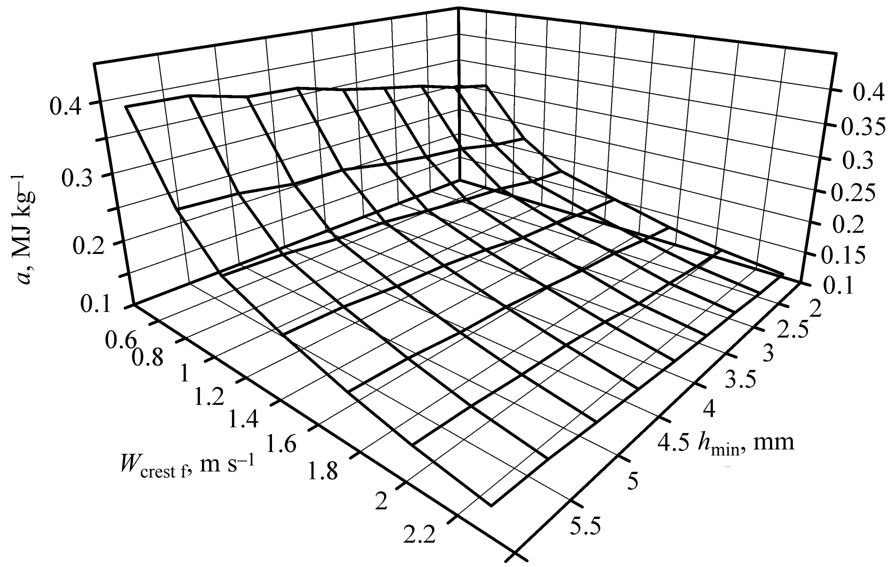

Fig. 6. The dependence of the specific mixing work on the depth of the minimum gap and speed of rotation of the fast rotor.

10 kW mm^{-1}), then at the speeds exceeding 1 m s^{-1} this dependence is more intense (about 50 kW mm^{-1}). Thus in the course of the operation of the fast mixer based on the power value consumed by the rotor drive we can conclude about a wear of the rotor crests

The mixing time upon a growth in the minimum gap value increases (Fig. 5) but unlike the character of changes in the power drive on $W_{\text{crest f}}$ and $h_{\min}$ and, at higher speeds of the processing the mixing time on the value of the minimum gap varies insignificantly.

One of the general indicators of the mixing process can be assumed the specific work of the mixer a (the work expended in mixing of 1 kg of the composition during its preparation) [7]. Fig. 6 shows the dependence $a = f_1(W_{\text{crest f}}, h_{\min})$, which has the character similar to the dependence of $t = f_2(W_{\text{crest f}}, h_{\text{crest}})$ (Fig. 5).

Since the surface of heat transfer of a mixer is defined as a result of its design, the heat calculation of the mixer must be verification. The purpose of this calculation is to determine the temperature of the composition, which should not exceed the permissible value [1, 9].

Usually in modern mixers with enhanced mode a time of the mixing cycle is less than the time during which a stable (equilibrium) temperature regime is installed, so the technique of heat calculation is based on the laws of nonstationary heat regime, when the temperature of the material increasing continuously. In this case, the task of calculation is to determine a relationship between the material temperature T_{m}, the coefficient of heat transfer k_{ht} from the material to the cooling water, and a cooling water flow. In this case, the finite time intervals, small enough to be able to use the arithmetic mean temperature difference, are considered at $\Delta T_{\text{f}i}/\Delta T_{\text{s}i} < 2$, where $\Delta T_{\text{f}i}$ and $T_{\text{s}i}$ are the temperature difference between the material and the cooling water at the beginning and end of the time interval Δt_i, K.

The mixing time is divided into n equal intervals Δt_i. The mixture temperature at the end of ith time interval is [1]

$$T_{\text{m}i} = \frac{Q_{\text{dis}i} - Q_{\text{r}i}}{M_{\Sigma} c_{\text{m}}} + T_{\text{m}i-1}, \qquad (20)$$

where $Q_{\text{dis}i}$ is the dissipation power for the time Δt_i ($Q_{\text{dis}i} = N_{\text{dr}}\eta_{\text{dr}}\Delta T_i$)), J; $Q_{\text{r}i}$ is the heat removed from the material for the time Δt_i with the help of the cooling water, J

$$Q_{\text{r}i} = k_{\text{ht}} S_{\text{mc}} (T_{\text{m}i} - T_{\text{r}i}) \Delta t_i,$$

where S_{mc} is an area of the heat transfer surface of the mixing chamber [with essential accuracy it can be assumed that $S_{\text{mc}} \approx 7\pi(R_{\text{crest}} + h_{\min})$], here $T_{\text{m}i}$ and $T_{\text{r}i}$ are the average temperature of the material and the cooling water in the course of time Δt_i.

After transformations Eq. 20 takes a form

$$T_{mi} = \left(1 - \frac{k_{ht} S_{mc} \Delta t_i}{M_\Sigma c_m}\right) T_{mi-1} + \frac{Q_{dis\,i}\, \Delta t_i}{M_\Sigma c_m} + \frac{k_{ht} S_{mc} \Delta t_i}{M_\Sigma c_m} T_{ws}, \qquad (21)$$

where T_{ws} is a starting temperature of the cooling water, K.

If we denote polynomials in Eq. (21) by A and B then this dependence for the first interval has a form [1]

$$T_{m1} = A + B T_{ms},$$

where T_{ms} is the starting material temperature in the course of the mixer loading, K; for the second interval:

$$T_{m2} = A + AB + B^2 T_{ms};$$

and for nth time interval:

$$T_{mn} = A(B^0 + B^1 + B^2 + \ldots + B^{n-1}) + B^n T_{ms}.$$

The material temperature T_{mn} at the end of the mixing should not exceed the permissible value that depends on the material properties.

Developed methods of parametric and thermal calculations of the rotary mixer enable us to determine the rational parameters and modes of operation of the equipment during the processing of thermoplastic materials.

NOTATIONS

a specific work, J kg^{-1};
c mass heat capacity, J kg^{-1} K^{-1};
G_V volume flow of material through a unit width of the gap, m^2 s^{-1};
h value pf gap, m;
k_{ht} coefficient of heat transfer from material to cooling water, W m^{-2} K;
K – coefficient of consistency, Pa s^n;
L length, m;
M weight, kg;
n exponent of the rheological equation;
N power, W;
p pressure, Pa;
q_V volume density of the heat flow, W m^{-3};
Q heat flow, W;
R radius, m;
S area, m^2;
t time, s;
T temperature, K;
V volume, m^3;
w, W linear speed, m s^{-1};
x, y, z Cartesian coordinates, m;
α heat transfer coefficient, W m^{-2} K^{-1};
$\alpha_{l\,(s)}$ angle of ascending helical line of the long (short) rotor blades, °;
$\dot{\gamma}$ shear rate, s^{-1};
ε dimensionless analog of coordinate x;
$\eta_{пр}$ factor of efficiency of the drive;
ξ dimensionless analog of coordinate y;
ρ density, kg m^{-3};
τ shear stress, Pa;
ω angular velocity of the working body, rad s^{-1}.
Main indexes:
0 for initial value;
i for ith element;
M for mass;
min for minimum value or minimum gap;
V for volume;
x, y, z for appropriate Cartesian coordinate;
f for fast rotor or final value;
w for cooling water;
crest for the crest;
dis for dissipation;
mc for the mixing chamber;
m for thermoplastic material;
s for starting value;
dr for drive;
r for operation surface of the rotor;
ss for the sickle-shaped gap;
sl for slow speed rotor.

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